A Transboundary Operating Model for Northeast Pacific Sablefish

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This work is ongoing; material presented here is preliminary and subject to change. To aid the reader, features which have yet to be developed in the model are designated by blue text, and the document concludes with a "near-term task" list.

Document and Project Description

Sablefish (*Anoplopoma fimbria*) are a highly mobile, long-lived, valuable groundfish that have high movement rates and range from Southern California to the Bering Sea. Synchronous sablefish population trends including a general decline across the entire range during the past few decades have increased concern about the populations' status. Traditionally, sablefish stock assessment and management has occurred independently at regional scales (Alaska, BC, US West Coast), assuming these are 'closed' stocks. However, recent genetic work has shown that NE Pacific sablefish are not genetically distinct between these traditional management areas, though there is evidence for differences in growth rate and size-at-maturity across the range. This suggests that the current delineation of assessment and management regions may be incongruent with the stock's actual spatial structure. In 2018, fisheries scientists from the US West Coast, Alaska, and British Columbia convened with the ultimate goal to construct a Management Strategy Evaluation (MSE) to evaluate the consequences of mis-specifying stock structure in assessment and harvest control rules. The first step of this project is the development of a spatially structured operating model integrating new movement and demographic information for the transboundary sablefish population, which is described and presented in this document.

This document describes the Operating Model (OM) developed for use in a Management Strategy Evaluation (MSE) for northeast Pacific Sablefish. The OM is coded in R (R Core Team, 2019) and Template Model Builder (Kristensen et al., 2016), and many elements are similar in structure to the widely-used statistical catch-at-age modeling software Stock Synthesis (Methot and Wetzel, 2013). The modeling framework was designed to represent several spatial areas, with movement occurring between spatial areas. The following sections describe the equations used to represent the population and fishery dynamics, and to condition the operating model.

Basic Operating Model Features

The operating model is age-structured, two-sex, and has an annual timestep (*y*). The plus-group age *A* for the model is 71 years; numbers- and biomass-at-age greater than age *A* are collapsed into this terminal group. Growth and selectivity are constant after this age. Here we provide a general description of the model dynamics through time. In this document, modeled spatial areas, termed sub-areas, are the union of biological stocks and political management regions (Figure 1). The biological stocks are defined by distinct demographic regimes, with growth described by Kapur et al. (2019), and movement among sub-areas by Rogers et al., (in prep). Sub-areas corresponding to the Alaskan Federal management regime are labelled "A"; sub-areas corresponding to British Columbia are labelled "B" and those off the West Coast of the United States/California Current labelled "C". In equations, sub-areas are indexed using the letters *i* and *j*, stocks using the letter *k,* and management regions using the letter *m.*

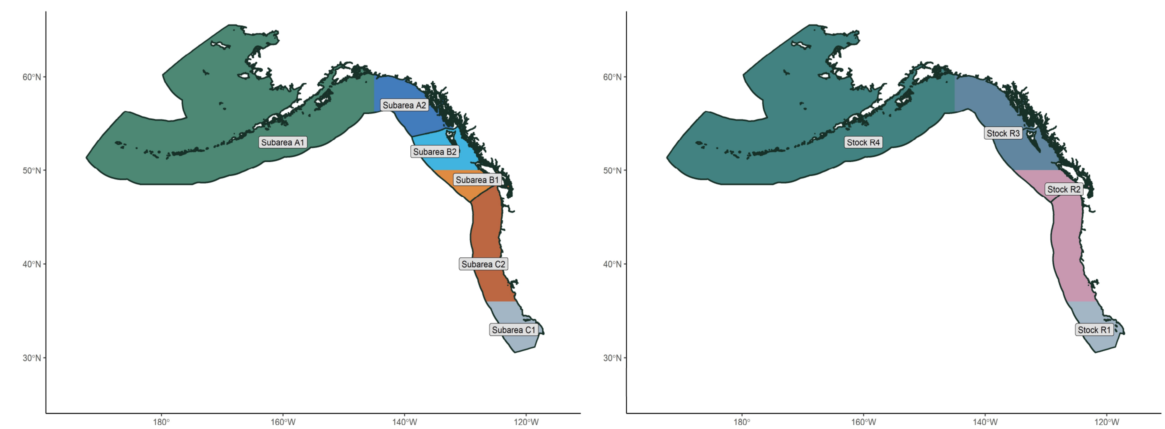

Figure 1. Schematic maps of sub-area and stock configuration with management regions. (Left) Map of spatial strata used in the operating model (shaded polygons with labels). Thick black lines delineate the current management regions. Sub-areas are referred to by the alphanumeric codes displayed on the map, with "A" referring to subareas within Alaska, "B" for those in British Columbia, and "C" for those in the California Current. Each sub-area corresponds to only one management region. (Right) Biological areas define the demographic regime, or "stock", occupied by the sub-areas. Figure colors in this document correspond to those shown here.

Population Dynamics (Numbers at Age)

The basic dynamics of fish of sex γ in year *y* at age *a* within sub-area *i* are given by:

$$N^i_{y+1,\gamma,a} = \begin{cases} 0.5R^i_{y+1} & if\ a = 0 \\ \sum_{i\neq j}\left[\left(1-\boldsymbol{X}^{i,j}_a\right)N^i_{y,\gamma,a-1}e^{-Z^i_{y,\gamma,a-1}} + \boldsymbol{X}^{j,i}_a N^j_{y,\gamma,a-1}e^{-Z^i_{y,\gamma,a-1}}\right] & if\ 1 \leq a < A \\ \sum_{i\neq j}\left[\left(1-\boldsymbol{X}^{i,j}_a\right)(N^i_{y,\gamma,A}e^{-Z^i_{y,\gamma,A}} + N^i_{y,\gamma,A-1}e^{-Z^i_{y,\gamma,A-1}}) \right. & if\ a = A \\ \left. \ldots + \boldsymbol{X}^{j,i}_a\left(N^j_{y,\gamma,A}e^{-Z^j_{y,\gamma,A}} + N^j_{y,\gamma,A-1}e^{-Z^j_{y,\gamma,A-1}}\right)\right] & \end{cases}$$

(1)

where $N^i_{y,\gamma,a}$ is the number of animals of age *a* and sex γ in sub-area *i* at the start of year *y*;

$\boldsymbol{X}^{i,j}_a$ is the age-specific matrix of movement probabilities from sub-area *i* to sub-area *j*;

$Z^{i}_{y,\gamma,a}$ is the total mortality experienced in sub-area *i* during year *y* for animals of age *a* given fishery mortality and stock-, sex- and age-specific natural mortality $M^{k}_{a,\gamma}$. See section on Catches for more detail on how total mortality is calculated;

R^{i}_{y} is the number of recruits to sub area *i* at the start of year *y* (see section on Reproduction); and

A is the maximum age (treated as a plus-group).

The model operates on a yearly timestep from 1960-2019, with the number of age-0 recruits by sex defined as half of the annual recruitment in each sub-area. There are implicit seasons in the model, where the "beginning" season corresponds to the start of the year. After fish grow and move among sub-areas, the population experiences half of the annual mortality and enters the "mid" season. At this time, fish are caught and surveyed by various fleets. The fishing mortality enacted by those fleets, plus the final half of annual mortality, is then applied to produce the "end" numbers at age, which are the figures represented by Equation 1. The numbers-at-age after movement and all mortality sources inform the spawning biomass and recruitment levels for the subsequent year. For ages 1 through the plus-group, the number of individuals in sub-area *i* of sex γ and age *a* at the start of the next year is the sum of individuals in sub-area *i* that survive and do not emigrate, or immigrate into sub-area *i* after surviving year *y* within another sub-area *j*. The plus group in sub-area *i* is comprised of individuals who age into or remain at age *A* in sub-area *i*, plus individuals who age into or remain at age *A* in sub-area *j* and move into sub-area *i*.

Growth

Length-at-age is stock- and sex-specific and follows a von Bertalanffy growth function (Bertalanffy, 1938), which is incremented for each timestep *y*. The incremental setup prohibits fish from shrinking if they move into a sub-area corresponding to a stock with a lower L_{∞}, because the growth increment is a function of the difference between current size and asymptotic size in the stock at hand. However, because fish are able to move between sub-areas with different growth patterns throughout their lifetime, there is a possibility that the mean length-at-age could decline in a given sub-area should a large influx of smaller-at-age fish enter (see below). We present the growth equations indexed to sub-area *i* though note that sub-areas nested within the same stock *k* share growth patterns. The mean length-at-age a at the start of year *y* for sex γ in sub-area *i* is given by:

$$L^{i}_{y+1,\gamma,a} = L^{i}_{y,\gamma,a} + (L^{i}_{\infty,\gamma} - L^{i}_{y,\gamma,a})\left(1 - e^{-\kappa^{i}_{\gamma}}\right)\ if\ a < A \quad (2)$$

For the plus group (age *A*) the mean size at the start of year *y* is calculated as the weighted average of fish entering and remaining in the plus-group within the sub-area. This calculation is based on the mean length-at-age in the middle of the (preceding) year. This setup allows the size of fish in the plus group to reflect changes (i.e. declines) due to exploitation and/or natural mortality (Methot & Wetzel, 2013).

$$L^{i}_{y+1,\gamma,A} = \frac{N^{i}_{y+1,\gamma,A-1}\left[L^{i}_{y,\gamma,A-1} + (L^{i}_{\infty,\gamma} - L^{i}_{y,\gamma,A-1})\left(1 - e^{-\kappa^{i}_{\gamma}}\right)\right] + N^{i}_{y+1,\gamma,A}\left[L^{i}_{y,\gamma,A} + (L^{i}_{\infty,\gamma} - L^{i}_{y,\gamma,A})\left(1 - e^{-\kappa^{i}_{\gamma}}\right)\right]}{N^{i}_{y+1,\gamma,A} + N^{i}_{y+1,\gamma,A-1}} \quad (3)$$

Where $\widetilde{L^{i}_{y,\gamma,a}}$ is the mean length-at-age at age a of sex γ in stock *k* at the midpoint of year *y*:

$$\widetilde{L^{i}_{y,\gamma,a}} = L^{i}_{y,\gamma,a} + (L^{i}_{\infty,\gamma} - L^{i}_{y,\gamma,a})\left(1 - e^{-0.5\kappa^{i}_{\gamma}}\right) \quad (4)$$

$L^{i}_{\infty,\gamma}$ is the stock and sex-specific asymptotic length (cm) for sub-area *i*; and

κ_{γ}^{i} is the stock and sex-specific growth rate (cm yr^{-1}) for sub-area *i*.

Unlike Stock Synthesis, all individuals present in a sub-area are subject to the growth pattern of the stock associated with that sub-area. This enables the modeling of ecosystem-based effects on the growth process, where a fish born in a southerly (slow-growing) region may grow to a greater size than expected based on its birth location if it moves to a northerly (fast-growing) region, and vice-versa.

Upon movement from one stock to another, the mean size of fish in the recipient sub-area becomes the weighted average of fish length-at-age already in the sub area and the length-at-age of fish entering the sub-area from a different sub-area *and* stock, both at the start of the previous year. In subsequent years, fish residing in sub-area *i* are subject to the same movement probabilities regardless of their natal sub-area; the growth pattern of recruits is not tracked nor fixed throughout their lives. This calculation takes place after the calculation of the mean size of the plus group (Equation 3) and therefore applies to all ages.

$$L_{y+1,\gamma,a}^{i} = \phi^{ij} \frac{N_{y+1,\gamma,a}^{i} L_{y,\gamma,a}^{i} + \sum_j N_{y+1,\gamma,a}^{j} L_{y,\gamma,a}^{j}}{N_{y+1,\gamma,a}^{i} + \sum_j N_{y+1,\gamma,a}^{j}} \qquad (5)$$

where ϕ^{ij} is a square array of *i* x *i* dimensions, indicating whether the new subarea *j* indeed belongs to

a different stock than the source sub-area *k*; ϕ^{ij} is 1 if the source and sink stocks for sub-areas i and j are distinct, and null otherwise. This structure means that if individuals move among sub-areas but not between *stocks* (which have distinct growth patterns), the weighted average is not calculated, and the sub-area retains the length-at-age values calculated in Equations 2-4.

$$\phi^{ij} = \begin{cases} \begin{array}{ccccccc} & A1 & A2 & B2 & B1 & C2 & C1 \\ A1 & \emptyset & 1 & 1 & 1 & 1 & 1 \\ A2 & 1 & \emptyset & \emptyset & 1 & 1 & 1 \\ B2 & 1 & \emptyset & \emptyset & 1 & 1 & 1 \\ B1 & 1 & 1 & 1 & \emptyset & \emptyset & 1 \\ C2 & 1 & 1 & 1 & \emptyset & \emptyset & 1 \\ C1 & 1 & 1 & 1 & 1 & 1 & \emptyset \end{array} \end{cases} \qquad (6)$$

The growth module generates a stock- and sex-specific matrix $\widetilde{\boldsymbol{L}}$ which defines the annual probability of being in each of *l* length bins at age *a* in the middle of the year. The mean length-at-age *a* for sex γ in sub-area *i* at the midpoint of the year, $\widetilde{L_{y,\gamma,a}^{i}}$, is used as an approximation to size-at-age for any samples collected during the year.

$$\widetilde{\boldsymbol{L}_{y,\gamma,a,l}^{i}} = \begin{cases} \Phi(\theta_1, \widetilde{L_{y,\gamma,a}^{i}}, \sigma_{G_{y,\gamma}}^{i}) & if\ l = 1 \\ \Phi(\theta_{l+1}, \widetilde{L_{y,\gamma,a}^{i}}, \sigma_{G_{y,\gamma}}^{i}) - \Phi(\theta_l, \widetilde{L_{y,\gamma,a}^{i}}, \sigma_{G_{y,\gamma}}^{i}) & if\ 1 < l < A_l \\ 1 - \Phi(\theta_l, \widetilde{L_{y,\gamma,a}^{i}}, \sigma_{G_{y,\gamma}}^{i}) & if\ l = A_l \end{cases} \qquad (7)$$

Similarly, the probability of being in each length bin at the start of year *y* is calculated using the length-at-age at the start of the year $L_{y,\gamma,a}^{i}$:

$$\boldsymbol{L}^{i}_{y,\gamma,a,l} = \begin{cases} \Phi(\theta_1, L^{i}_{y,\gamma,a}, \sigma^{i}_{G_{y,\gamma}}) & if\ l = 1 \\ \Phi(\theta_{l+1}, L^{i}_{y,\gamma,a}, \sigma^{i}_{G_{y,\gamma}}) - \Phi(\theta_l, L^{i}_{y,\gamma,a}, \sigma^{i}_{G_{y,\gamma}}) & if\ 1 < l < A_l \\ 1 - \Phi(\theta_l, L^{i}_{y,\gamma,a}, \sigma^{i}_{G_{y,\gamma}}) & if\ l = A_l \end{cases} \tag{8}$$

where Φ is the normal cumulative density function, which is evaluated for θ_l given a mean and standard deviation;
θ_l is the lower limit of length bin *l*;
A_l is the index of the largest length bin;
$\sigma^{i}_{G,y,\gamma}$ is the stock and sex-specific standard deviation of length-at-age, which is time blocked for
some sexes (Table 3).

Specific values for the growth parameters are shared among sub-areas from the same stock as shown in Table 1. Body weight is converted from length via:

$$w^{i}_{\gamma,l} = \alpha^{i}_{\gamma} \bar{L}_l^{\beta^{i}_{\gamma}} \tag{9}$$

where α^{i}_{γ} and β^{i}_{γ} are stock- and sex- specific constants of the allometric length-weight equation; and
$\bar{L}_l$ is the midpoint of the population length bin.

The population-level body weight-at-age in a stock (or sub-area) at the start of the year is calculated using the stock- and sex-specific weight-at-age and the proportions at length at the start of the year:

$$w^{k}_{y,\gamma,a} = \sum_l \boldsymbol{L}^{i}_{y,\gamma,a,l} w^{i}_{\gamma,l} \tag{10}$$

The same calculation can be done using the mid-year stock- and sex-specific weight-at-age and proportions at length.

$$\tilde{w}^{k}_{y,\gamma,a} = \sum_l \tilde{\boldsymbol{L}}^{i}_{y,\gamma,a,l} w^{i}_{\gamma,l} \tag{11}$$

Reproduction or Recruitment

Recruitment follows a Beverton-Holt (1957) stock-recruitment curve with annual deviations. Density-dependence is assumed to occur at the level of the stock (*k*) and is determined by the biomass (converted from numbers, as described above) of mature females in the stock, which may be the sum across two or more sub-areas, at the start of year *y*. The stock-recruitment relationship assumes that recruitment deviates occur at the stock level (and are thus equivalent among sub-areas within a stock).

$$R^{k}_{y} = \frac{4h^k R^k_0 S^k_y}{S^k_0(1-h^k) + S^k_y(5h^k-1)} e^{-0.5\sigma_R^2 + \tilde{R}^k_y} \tag{12}$$

where h^k is the steepness of the stock recruitment curve (expected proportion of R_0 at $0.2S_0$) for stock *k*;
R^k_0 is the virgin recruitment for stock *k*;
$\tilde{R}^k_y$ are random annual recruitment deviations specific to stock *k* and assumed to be normally distributed with mean zero and standard deviation σ_R;

S_y^k is the spawning biomass (mature females) in stock k at the start of year y. Recall that there is never more than one stock k in a sub-area i:

$$S_y^k = \sum_i \sum_a \phi^{ik} N_{y,\gamma=female,a}^i w_{y,\gamma=female,a}^k E_a^k \quad (13)$$

ϕ^{ik} is a matrix with values set to 1 if sub-area i is nested within stock k, and 0 otherwise;

$$\phi^{ik} = \begin{cases} & R1 & R2 & R3 & R4 \\ A1 & 0 & 0 & 0 & 1 \\ A2 & 0 & 0 & 1 & 0 \\ B2 & 0 & 0 & 1 & 0 \\ B1 & 0 & 1 & 0 & 0 \\ C2 & 0 & 1 & 0 & 0 \\ C1 & 1 & 0 & 0 & 0 \end{cases} \quad (14)$$

E_a^k is the proportion of females at age in stock k that have reached maturity at age a OR the fecundity at age, which is the integral over the length of the product of weight-at-length (like the weight equation, but instead of averaging over weight-at-length we average over proportion mature-at-length); and

S_0^k is the unfished female spawning biomass of stock k (Equation 28).

Recruits are allocated from stock k to sub-area i based on a fixed proportion. The resulting annual recruitment spawned within each sub-area is used in Equation 1.

$$R_y^i = \tau_y^{ik} e^{\epsilon_\tau} R_y^k \quad (15)$$

were τ_y^{ik} is a user-defined matrix specifying the distribution of recruits from stock k to sub-area i; by definition, there will be spatial autocorrelation in the number of recruits in sub-areas within the same stock - the year subscript is used to allow for variation in the proportions through time; and

ϵ_τ is a lognormally-distributed variance term for τ_y^{ik}.

Fleets

A fleet is a discrete survey or fishing operation. There are several fleets *f* in each management region *m*, and there may be more than one fleet operating in each sub-area (Figure 2). All management regions have coverage by at least one fishery fleet or survey, and have length- or age-composition data from one or more fleets. Temporal coverage varies by management region, with catch records extending back to the early 1900s for Alaska and the California Current (Table 1). The Operating Model's main period, however, starts in 1960.

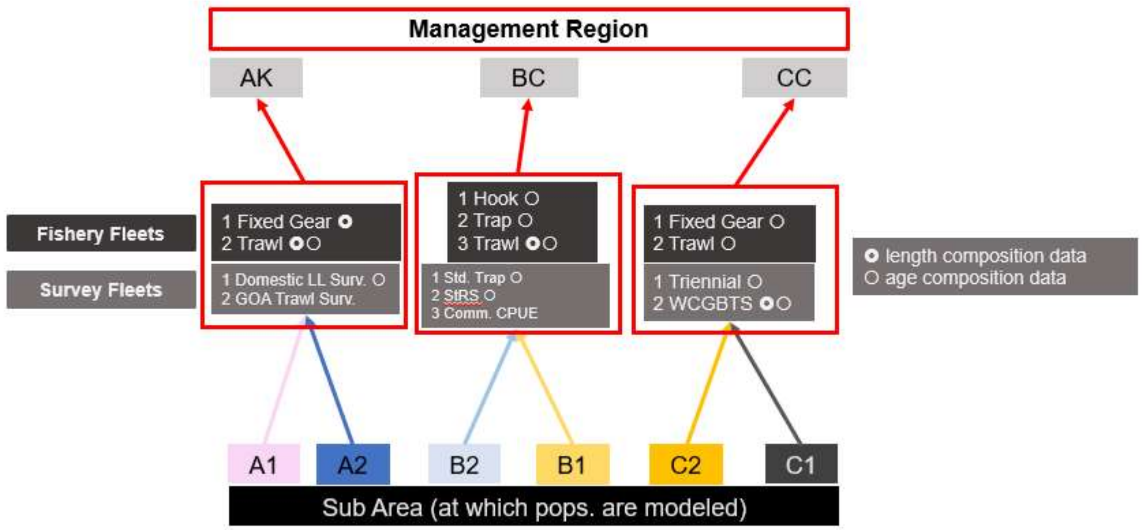

Figure 2. Schematic of available survey fleets (light grey boxes) fishery fleets (dark grey boxes) and associated compositional data (circular symbols) available for each of three management regions (lightest grey boxes at top). The sub-areas are fished/sampled by the fleets to which colored arrows point. Note that indices of relative abundance were re-created using a spatio-temporal model that reduced the number of survey fleets for several regions; compositions were retained at original fleet resolution.

Catches

Catches accrue to one of three management regions, which are comprised of at least one sub-area *i*. The data used to condition and test this OM are aggregated at the fleet level *f*, which are nested within management regions *m* and may exploit more than one sub-area *i*.

Discards

At present, discard data is not yet included in the model.

Total fishing mortality reflects both retained fish (according to a retention function) and the fraction of catches that are discarded and assumed to die. Ω is a logistic function defining the fraction of retained catch of animals of age *a*, during year *y* for fishery fleet *f* and sex γ. The function for discarded catch is simply 1- Ω. The mortality of discarded catch may vary by the age of fish.

$$\Omega^{f}_{y,\gamma,a} = \beta_3^{y,f,\gamma,a}\left(1 + exp(-a - (\frac{\beta_1^{y,f,\gamma,a} + \beta_4^{y,f,\gamma,a}}{\beta_2^{y,f,\gamma,a}})\right)^{-1} \quad (16)$$

Where $\beta_1^{y,f,\gamma,a}$ is the age at the inflection point of the logistic retention function;

$\beta_2^{y,f,\gamma,a}$ is the slope at the point of inflection;
$\beta_3^{y,f,\gamma,a}$ asymptotic fraction retained (note that discards + retained must sum to 1);
and
$\beta_4^{y,f,\gamma,a}$ is 0 for females and an arithmetic offset for male ages.

Retained Catch

Fleet- and age-specific fishery catch is modeled using the Baranov catch equation (Ricker, 1975). The annual retained catch for each fleet in each management region $C_y^{m,f}$ is obtained by summing the catches by fleet *f* across all sub-areas *i* that are both fished by fleet *f* and nested within management region *m*. No fleet fishes in more than one management region. The use of the Baranov equation ensures that the numbers-at-age in any given year are always greater than zero.

$$C_y^{m,f} = \sum_{i\epsilon m} \sum_\gamma \sum_a \phi^{if} w_{\gamma,a}^f \frac{s_{\gamma,a}^f F_y^f \Omega_{y,\gamma,a}^f}{Z_{y,\gamma,a}^i} N_{y,\gamma,a}^i (1 - e^{-Z_{y,\gamma,a}^i}) \quad (17)$$

Where ϕ^{if} is a matrix indicating whether fleet *f* occurs in sub-area *i (ϕ^{if}* is set to 1 if fleet *f* occurs in
sub-area *i*, and 0 otherwise);
$w_{\gamma,a}^f$ is fleet- and sex- specific weight-at-age for captured fish (assumed equivalent for all sub- areas *i* fished by *f*);
$s_{\gamma,a}^f$ is the selectivity by fleet *f* for animals of age *a* and sex γ (see section on Selectivity);
F_y^f is the fishing mortality rate on fully recruited animals by fleet *f* during year *y* (described in more detail below);
$Z_{y,\gamma,a}^i$ is the total fishing mortality enacted by all fleets on fish of sex γ and age *a* in sub-area *i* during year *y*. As in Equation 1, we assume that F_y^f is equivalent across sub-areas (i.e. the fishing mortality by fleet *f* in sub-area *i* is the same as the fishing mortality of fleet *f* by sub-area *j*) :

$$Z_{y,\gamma,a}^i = M_{\gamma,a}^i + \sum_f \phi^{if} (s_{\gamma,a}^f F_y^f \Omega_{y,\gamma,a}^f + s_{\gamma,a}^f F_y^f (1 - \Omega_{y,\gamma,a}^f) V_a^f) \quad (18)$$

V_a^f is the fleet- and age-specific mortality rate of discarded fish; if set to zero, all discards are assumed to survive and only retained catches figure into total mortality.

Fishing Mortality

The annual fishing mortality due to each fleet F_y^f is determined using the "hybrid" method, which has been implemented in Stock Synthesis as well as other stock assessment packages. The premise behind the hybrid method arises from the fact that F_y^f cannot be solved for explicitly but would be computationally expensive to estimate. Instead, the method algorithmically tunes the continuous *F* values for each fleet until the observed catches by fleet (across one or more sub-areas, as necessary) are matched. This is preferable to Pope's approximation, which is invoked to provide the starting values for the tuning algorithm. The steps in the hybrid approach are as follows.

1) Identify an initial guess $\tilde{F}_y^{f1}$ for the annual fishing mortality of each fleet *f*. This initial guess is the ratio of the fleet's observed catch during year *y* to the total exploitable biomass available

to fleet *f* at the start of the year *y*. In the case where fleet *f* fishes more than one sub-area, the total exploitable biomass (denominator) necessarily sums biomass from each sub-area *i* where fleet *f* is active. Therefore, the resultant $\tilde{F}_y^{f1}$ corresponds to the entirety of fleet *f's* exploitation:

$$\tilde{F}_y^{f1} = \frac{C_{obs,y}^f}{\sum_i \sum_\gamma \sum_a \phi^{if} w_{a,\gamma}^f s_{a,\gamma}^f N_{y,\gamma,a}^i + C_{obs,y}^f} \tag{19}$$

where C_{obs}^f is the observed catch (retained + discard) for fleet f.

2) This initial guess is modified to become the starting value F_y^{f1} following Pope's approximation:

$$F_y^{f1} = -ln\left(1 - \left[\tilde{F}_y^{f1}\left(\frac{1}{1+exp(30(\tilde{F}_y^{f1}-v)}\right) + v(1 - \left(\frac{1}{1+exp(30(\tilde{F}_y^{f1}-v)}\right)\right]\right) \tag{20}$$

where v controls the upper limit on F_y^{f1}, as $\tilde{F}_y^{f1} \rightarrow \infty, F_y^{f1} \rightarrow -ln(1-v)$. This was set to 0.7, which corresponds to $F_y^{f1} = 1.5$. In other words, harvest rates above 0.7 are converted to an *F* corresponding to a harvest rate close to 0.7.

3) Compute model-predicted catches for each fleet. This includes a summation of biomass across all sub-areas fished by *f*, subset via ϕ_{if}. At this point, $Z_{y,\gamma,a}^i$ is the total mortality in sub-area *i* implied by the current guesses for fishing mortality F_y^{f1}, as in Equation 19. If there is more than one fleet present in *i*, $Z_{y,\gamma,a}^i$ incorporates the current guesses for fishing mortality for all fleets.

$$C_{pred,y}^f = \sum_i \sum_\gamma \sum_a \phi^{if} w_{\gamma,a}^f \frac{s_{\gamma,a}^f \Omega_{y,\gamma,a}^f F_y^{f1}}{Z_{y,\gamma,a}^i} N_{y,\gamma,a}^i (1 - e^{-Z_{y,\gamma,a}^i}) \tag{21}$$

4) Compute an adjustment factor *Adj* that will be used to tune total mortality. *Adj* is calculated using the ratio of the sums of observed and predicted catches from all fleets *f* operating within sub-area *i*:

$$Adj_{\ y}^{i} = \sum_f \phi^{if} \frac{C_{obs,y}^f}{C_{pred,y}^f} \tag{22}$$

5) Re-scale total mortality in sub-area *i* using $Adj_{\ y}^{i}$ and the extant guess for fishing mortality:

$$\tilde{Z}_{y,\gamma,a}^i = M_{\gamma,a}^i + \sum_f \phi^{if} Adj_{\ y}^{i} (s_{\gamma,a}^f F_y^{f1} \Omega_{y,\gamma,a}^f + s_{\gamma,a}^f F_y^{f1}(1 - \Omega_{y,\gamma,a}^f) V_a^f) \tag{23}$$

6) For the next iteration, the value of F_f is given by updating C_{pred}^f (Equation 20) with the new $\tilde{Z}_{y,\gamma,a}^i$ and repeating steps 2-4. The new F_f "guess" represents the ratio between the observed catches for fleet *f* and total exploitable biomass available to fleet *f*, given the adjusted value for total mortality found in Step 5.

$$\widetilde{F_y^{f2}} = \frac{C_{obs}^f}{\sum_i \sum_\gamma \sum_a \phi^{if} w_{\gamma,a}^f \frac{s_{\gamma,a}^f \Omega_{y,\gamma,a}^f F_y^{f1}}{\tilde{Z}_{y,\gamma,a}^i} N_{y,\gamma,a}^i (1 - e^{-\tilde{Z}_{y,\gamma,a}^i})} \tag{24}$$

7) This $\widetilde{F_y^{f2}}$ is again modified following Equation 20, with the change that *v* is multiplied by *Fmax*, here set to 1.5. This ensures that as $\widetilde{F_y^{f2}}$ (or any subsequent iterations) approaches 0. 7Fmax, F_y^{f2} approaches Fmax.

$$F_y^{f2} = -ln\left(1 - \left[\widetilde{F_y^{f2}}\left(\frac{1}{1+exp(30(\widetilde{F_y^{f2}}-vF_{max})}\right) + v(1 - \left(\frac{1}{1+ex\ \ (30(\widetilde{F_y^{f2}}-vF_{max})}\right)\right]\right) \quad (25)$$

8) Steps 2-7 are repeated several times. The final iteration terminates with a value of F_y^f following Equation 25, which is applicable to the entire exploitation activity of a given fleet across a management region.

Selectivity (in Fleets & Surveys)
Selectivity is both age- and length-specific in the operating model. For fleets operating in AK and CC management regions, the length-based selectivity function is 1.0 for all fleets and lengths as these assessments have historically not modeled length-based selectivity for any fleet. Aside from this constant (fully-selected) setup, selectivity curves can follow an asymptotic or "dome-shaped" selectivity pattern, the latter described by either a normal or gamma distribution. At present, all survey fleets follow a logistic age-based selectivity pattern, and only fishery fleets or compositional survey fleets from British Columbia use length-based selectivity (Table 6).

The asymptotic selectivity curve follows

$$s_a^f = (1 + e^{-(a-a_{50}^f)/\delta^f})^{-1} \quad (26)$$

where a_{50} is the age at 50% selectivity for the logistic (asymptotic) curve; and
δ is a parameter of the asymptotic curve; and

Fleets which obtained very high estimates for the terminus of the descending limb were converted to asymptotic selectivity curves.

Movement
Movement between areas is modeled using a matrix **X**, which represents the transition probability of fish at age *a* in sub-area *i* at the time before catch is removed and surveys are conducted. The rows of the matrix correspond to the sub-areas in Figure 1, and the column headers correspond to the sub-areas which consists of up to *A* matrices with elements representing the proportion of fish at age a in area i which move to another area *j* at the time when the catch is removed / when the surveys are conducted. For simulations in which movement is "off", all off-diagonal values of **X** are set to zero and diagonal elements are set to one; Movement parameters were obtained by the analysis of several decades' tag-recapture data for sablefish, implemented here as a saturating function of age. The resultant movement-at-age values among sub-areas for each sex are presented in Figure 8.

Equilibrium Abundance
To initialize the model, we calculate the unfished age distribution at the stock level and partition into sub-areas based on the following. We generate these values by running the following equations for many years (i.e. 10*A*).

$$N_{0,\gamma,a}^{i} = \begin{cases} 0.5R_0^k \tau^{ik} e^{\epsilon_{\tau^{ik}} - \sum_a M_a^k} & if\ a < A \\ \frac{N_{\gamma,A-1}^{i} e^{-\sum_a M_a^k}}{1-e^{-M_a^k}} & if\ a = A \end{cases} \quad (27)$$

where ω_a^i is the solution to the unfished numbers-at-age matrix described in the Movement section, with an entry for each sub-area within each stock.

τ^{ik} is a user-defined matrix specifying the distribution of recruits in stock *k* to sub-area i.

Unfished equilibrium stock spawning biomass is calculated via:

$$S_0^k = \sum_i \sum_a \phi^{\text{ik}} N_{0,\gamma=female,a}^{i} w_{0,\gamma=female,a}^{k} E_a^k \quad (28)$$

Initial Conditions

The model is first run with *A* years (denoted "init") of movement, recruitment deviations, and without fishing:

$$N_{init,\gamma,a}^{i} = \begin{cases} 0.5\tau^{ik} R_0^k e^{-\sum_a M_a^k} e^{-0.5\sigma_R^2 + \tilde{R}_{init-}^k} & if\ a < A \\ \frac{N_{init-\ ,\gamma,A-1}^{i} e^{-\sum_a M_a^k}}{1-e^{-M_a^k}} e^{-0.5\sigma_R^2 + \tilde{R}_{init-}^k} & if\ a = A \end{cases} \quad (29)$$

Data Generation

Surveys

Surveys occur at the midpoint of the year. The operation of some survey fleets span more than one sub-area, in which case expected survey biomass B_y^f is computed as the sum across sub-areas (the assumption of homogeneity within stocks is held). For fleets that do not record the ages of surveyed fish, the *a* subscript on *s* is ignored.

$$B_y^f = \epsilon_y^f q^f \sum_i \sum_a \phi^{if} w_a^{k,f} s_a^f N_{y,a}^i \quad (30)$$

where ϵ_y^f is the survey fleet- and year-specific error term, drawn from a lognormal distribution with a standard error of the logarithm based on standard deviation in log-space of 0.2 (Francis, 2011) added to a fleet-specific variance specific to survey years $\sigma_{S,y}^f$, calculated externally to the model:

$$\epsilon_y^f \sim lognormal(0,0.2 + \sigma_{S,y}^f) \quad (31)$$

q^f is the catchability coefficient for survey fleet *f*;

ϕ^{if} is a matrix defining whether survey fleet *f* operates in sub-area *i*; and

s_a^f is the selectivity for survey fleet *f*, which may follow one of three functional forms (see below).

Length Compositions

Fishery and survey length compositions π are generated by sex per year for applicable fleets based on the landed catch ($C_{y,\gamma,l}^f$). For the fisheries, the proportion of individuals at length in the catch for fleet *f* in year *y* is found by dividing the prevalence in catch at length by the total caught biomass at length.

$$\pi^f_{y,\gamma,l} = \frac{C^f_{y,\gamma,l}}{\sum_a C^f_{y,\gamma,l}} \quad (32)$$

Aging Error

The expected proportion of observed numbers-at-age must account for imprecision and bias in aging error due to otolith reads occurring at different labs and/or by different readers. The input aging error matrix converts true ages a into expected ages $\tilde{a}_a$, which are used to index the expected age value at age a and sex γ at the midpoint of year y (when samples are obtained). The aging error matrix is estimated external to the model.

$$\pi^f_{y,\gamma,a} = \begin{cases} \Phi(\theta_1, \tilde{a}_{a,\gamma}, \sigma^f_{A_\gamma}) & if\ a = 1 \\ \Phi(\theta_{a+1}, \tilde{a}_{a,\gamma}, \sigma^f_{A_\gamma}) - \Phi(\theta_a, \tilde{a}_{a,\gamma}, \sigma^f_{A_\gamma}) & if\ 1 < a < A \\ 1 - \Phi(A, \tilde{a}_{a,\gamma}, \sigma^f_{A_\gamma}) & if\ a = A \end{cases} \quad (33)$$

where $\pi^f_{y,\gamma,a}$ is the proportion of animals of true ages a of sex γ observed to be $\tilde{a}$ from survey fleet f at
the midpoint of year y;
$\tilde{a}_a$ is the expected age a incremented to mid-year values by adding 0.5; and
Φ is the normal cumulative density function; and
θ_a is the lower limit of age bin a; and
$\sigma^f_{A\ \gamma}$ is the fleet-, age and sex-specific standard deviation of observed ages. Generally, these are
shared among fleets in management regions.

Collapsing Surveys & Compositional Data

Some estimation methods required combining data over multiple sub-areas. Converting survey index information from individual fleets and years B^f_y into a single expected index is achieved using a summation across fleets among years:

$$B^{f'}_y = q^{f'} \sum_i \sum_y \sum_a \phi^{if} s^f_a N^i_{y,a} w^f_{y,a} \quad (34)$$

To combine length- and/or age-compositions from multiple surveys, the fleet-specific proportions π^f_a are weighted by the total observed numbers at age in sub-areas exploited by fleet f in that year $\tilde{N}^i_{y,\gamma,a}$, which is calculated within the OM. These weighted values are then summed across fleets for each year and divided by the total number observed in all sub-areas concerned, returning the proportions-at-age for a single survey combining multiple fleets.

$$\pi^{f'}_{y,a} = \frac{\sum_{f\epsilon f'} \sum_i \sum_\gamma \phi^{if} \pi^f_a \tilde{N}^i_{y,\gamma,a}}{\sum_i \sum_\gamma \tilde{N}^i_{y,\gamma,a}} \quad (35)$$

Reference Points

Reference points are to be calculated for each management region but are not yet available in the present model. The derivation of reference points, and projection of the model forward (for experimentation with various harvest control rules, for example) relies upon the fishery selectivity patterns and relative fishing intensity among fleets (F^f). The fully-selected fishing mortality corresponding to *MSY*, F_{MSY}, is defined as the instantaneous rate of fishing mortality at which yield is maximized, is obtained via the following. Since dynamics happen at the stock and/or sub-area level, summations are made as necessary to obtain reference points at the scale of management.

1) For each fleet within management region *m*, calculate the time-averaged selectivity $\widehat{s^f_{\gamma,a}}$ and biology (body weight-at-age $\widehat{w^f_{\gamma,a}}$), and the relative fishing intensity per fleet $\widehat{F^f}$.
2) Define recruitment in the management area as a function of *F*, based on the stock-recruitment relationship defined for the OM. The numbers-at-age are the equilibrium abundances from Eq 28

$$R^m = \frac{\sum_{k\epsilon m}\sum_{f\epsilon m}\sum_a \phi^{if}\widehat{F^f} N^i_{\gamma=female,a} - \sum_{k\epsilon m}\alpha^k}{\beta^k\sum_{k\epsilon m}\sum_{f\epsilon m}\sum_a \phi^{if}\widehat{F^f} N^i_{\gamma=female,a}} \qquad (36)$$

Where $\alpha^k = S^k_0\left(\frac{1-h^k}{4h^k}\right)$;

$S^k_0 = 0.5\sum_a E^k_a R^k_0 e^{-M^k_a}\,\widehat{w^f_{\gamma=female,a}}$; and

$\beta^k = \frac{5h^k-1}{4h^k R^k_0}$

3) Define yield-per-recruit for the entire management region, as a function of all fleets contained within the region and the biomass of sub-areas fished by those fleets. As this set-up aims to maximize yield, not total deaths, only retained catches are included:

$$\tilde{Y}^m = \sum_{f\epsilon m}\sum_\gamma\sum_a \phi^{if}\widehat{w^f_{a,\gamma}}\frac{\widehat{s^f_{\gamma,a}}\widehat{\Omega^f_{\gamma,a}}\widehat{F^f}}{Z^i_{\gamma,a}} N^i_{\gamma,a}\left(1-e^{-Z^i_{\gamma,a}}\right) \qquad (37)$$

where $N^i_{a,\gamma}$ is the number of fish of sex γ and age *a* relative to the total number (both sexes) of age-zero fish, given *F*, in equilibrium:

$$N^i_{a,\gamma} = \begin{cases} 0.5 & if\ a = 0 \\ N^i_{a-1,\gamma}e^{-Z^i_{\gamma,a-1}} & if\ 0 < a < A \\ \frac{N^i_{a-1,\gamma}e^{-Z^i_{\gamma,A-1}}}{1-e^{-Z^i_{\gamma,A}}} & if\ a = A \end{cases} \qquad (38)$$

$Z^i_{\gamma,a}$ is the total mortality present in sub-area *i:*

$$Z^i_{\gamma,a} = M^i_{\gamma,a} + \sum_f \phi^{if}\left(\widehat{s^f_{\gamma,a}}\widehat{F^f}\,\Omega^f_{\gamma,a} + \widehat{s^f_{\gamma,a}}\widehat{F^f}\left(1-\Omega^f_{\gamma,a}\right)V^f_a\right) \qquad (39)$$

4) Define the "yield function" as the product of yield per recruit under fully-selected fishing mortality and recruitment:

$$Y^m = \tilde{Y}^m R^m \qquad (40)$$

5) Solve for the level of F that, when multiplied by the relative fishing intensities found in Step 1, maximize the yield function, i.e.:

$$\left.\frac{dY^m}{dF^m}\right|_{F^m_{MSY}} = 0 \qquad (41)$$

Other Monitored Quantities

Unfished Biomass

The OM monitors equilibrium-based and dynamic indicators of stock status, known as static or dynamic B_0, respectively. Static B_0 is the deterministic equilibrium biomass prior to fishing, also known as the unfished biomass, and is the typical quantity used for reference point calculation and is presented in Equation 28. Dynamic B_0 is defined annually as the expected spawning biomass in the absence of fishing mortality F. This is achieved by re-evaluating Equation 1 with the F component of total mortality Z set to zero, and converting numbers-at-age to biomass-at-age using Equation 13.

In addition to the reference points described above, the model reports B_{low} (the lowest spawning biomass ever encountered) and F_{crash} (the lowest fishing mortality corresponding to equilibrium biomass of zero).

Model Projections (Forecasts)

Forecasting is not yet implemented.

Forward projection of the model beyond the period with catch and survey data necessitates the following assumptions, which can be divided between process and observational components. Catches and observed survey biomass for each fleet are fixed to the average of the previous five years. Observed length and/or age compositions are bootstrapped resampling from the previous five years for each fleet. Selectivity patterns for all fleets are identical to the terminal year of the simulation. Demographic values remain identical to the terminal year of the simulation, meaning that process error in forecasted years is like the main model period. If a recruitment bias adjustment ramp is implemented, the bias correction factor b_y for forecasted years is fixed to 0.5, which leads to a median ratio of present to virgin spawning biomass of approximately 1.

Operating Model Data Inputs and Treatment

Input parameters, estimation boundaries and data used in conditioning the OM are available in Tables. Below we describe the data sources for the OM and treatment thereof.

Demographic Parameters

Growth

The spatial structure of "stocks" (red dashed lines, Figure 1) are based upon the growth analyses performed by Kapur et al. (2020), which identified five unique regions of sablefish growth corresponding to major oceanographic features. In brief, growth in the OM follows a latitudinal cline whereby sablefish obtain a higher asymptotic length L_∞ at more north-western locales (i.e. sub-areas A1 and A2). The OM also has a time block in growth for females for all but the most north-westerly sub-areas, following findings by Kapur et al. (2020) that significant differences in growth parameters are present in females in these sub-areas before and after 2010. The results presented in Kapur et al. (2020) were updated to re-estimate individual values for σ_G.

Maturity
Stock-specific maturity was estimated using both macroscopic and histological data for the three management regions at the stock strata (Williams et al., in prep). The estimates were obtained using a generalized additive model which had covariates for depth, age, length, region, and sampling location. This model was identified among other candidate models using AIC. The resultant maturity-at-age curves for each stock are shown in with parameters thereof presented in Figure 9

Movement
Movement among sub-areas was estimated external to the OM using an analysis of over 30 years of tag-recapture data, and provides movement parameters for sablefish between 400-800mm among the 6 modeled sub-areas (Rogers et al., in prep). Because our model is length-based, we performed a simple conversion between these pooled movement estimates and sex-, stock-, and age-specific size. We used the deterministic (expected) length-at-age curve for each sub-area to determine at which age fish in that sub-area are expected to reach 400mm in length, which of course varied by sex and stock. All sexes and sub-areas are expected to be above 400m by age 5, for which the movement matrix is fully populated; for age 4, only fish in a subset of sub-areas are expected to reach the 400m threshold, so the movement matrix is sparse for certain sex-sub-area combinations at age 4. We assumed that movement was zero before age 4, and the upper limit of 800mm captures our length plus-group for all sub-areas. The resultant movement matrices are presented in Figure 8. We plan to implement estimation of recruitment distribution within stocks, which should account for the movement of age-0 fish at the stock scale, and potentially estimate juvenile movement (ages 1-4).

Fleets and Catches
The available fleets and compositional data corresponding to each region are shown in Figure 2; coverage by sample size and year is shown in Figure 3, and actual catch values used in the model are shown in Figure 4.

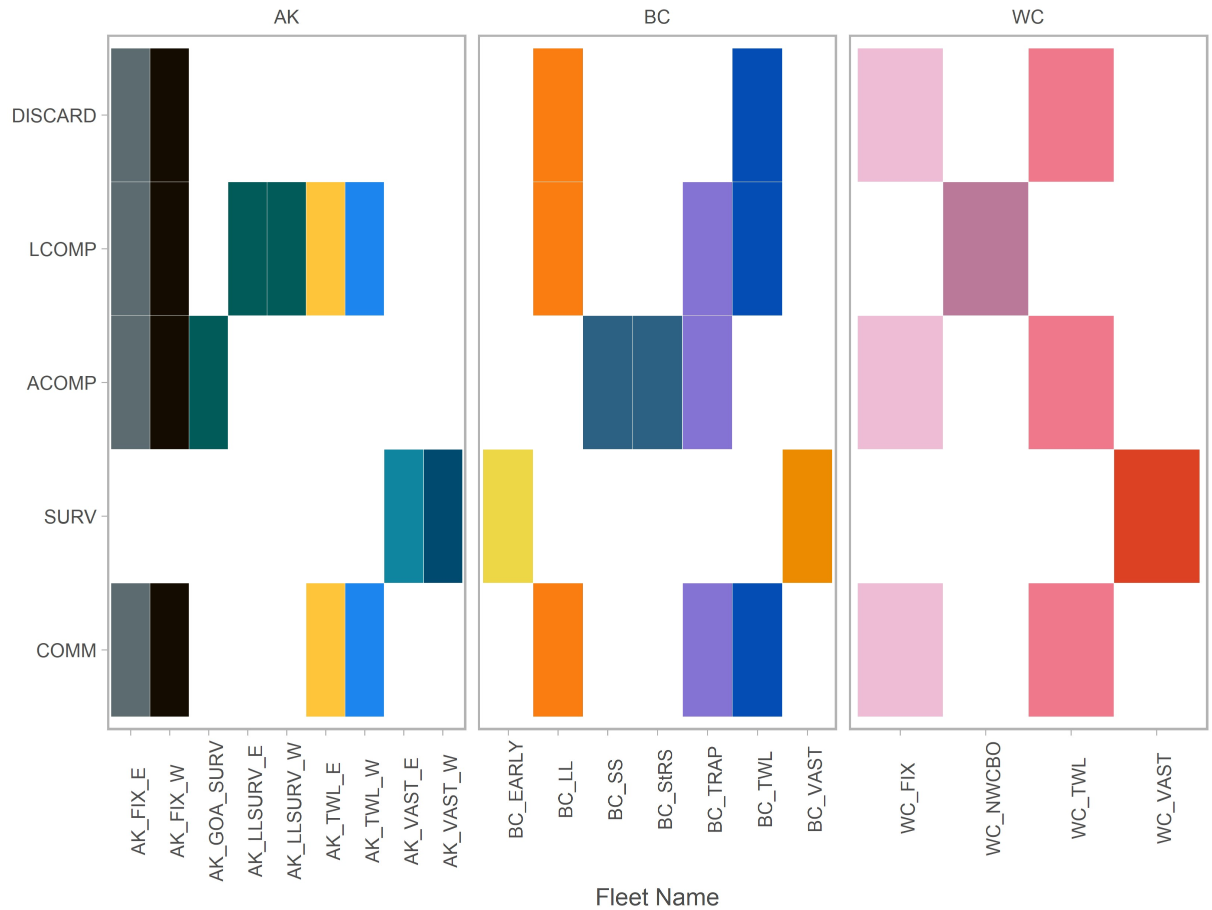

Figure 3. Plot of data to be included by source in Operating Model. Available data is colored by data type and paneled by management region (AK = Alaska, BC = British Columbia,WC = US West Coast/California Current). Note that at present, discards and length-comp data are not yet used in the operating model. While length data are available for both fisheries and surveys from the WC, we are not planning to include commercial length information, which is in keeping with the recent assessment (Haltuch et al., 2019).

Fishery Catches

There are at least two fishery-dependent sources of catch records from each management region. For the CC, the hook and line and pot fisheries were combined into a single fixed gear-fishery. Alaskan commercial fleets (FIX and TWL) are each separated into time series east and west of 145°W for use in the model, but for confidentiality values are aggregated for all of Alaska.

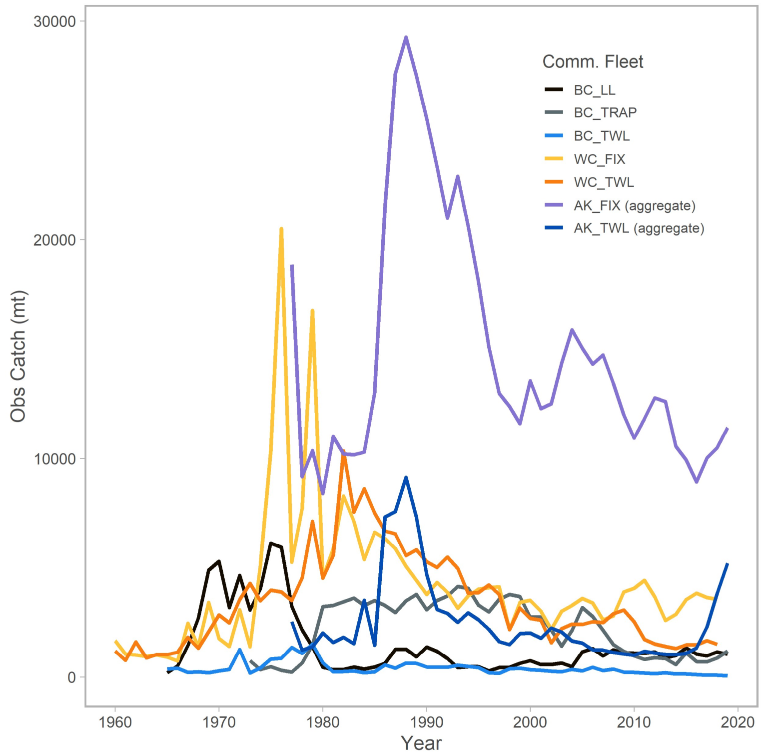

Figure 4. Fishery catches in mt used in operating model. Note that AK FIX and TWL fleets are split east and west of 145°W for use in the OM, but aggregated here for confidentiality reasons.

Fishery Discards

Discards are not yet implemented in the operating model.

The treatment of discarded catch varies by fleet and management region; the mortality rates V were fixed according to values from regional assessments for use in this OM. For the CC, discard mortality is 20% for sablefish caught with fixed gear and 50% for sablefish captured with trawls, except for age-0 fish which were assumed to experience 100% discard mortality. For BC fleets, discard mortality is assumed to be 15% for trap fleets, 30% for longline hook and 80% for trawl gears (Cox et al., 2011). In Alaska, discards estimates were unavailable before 1993 (Hanselman et al., 2019) yet always account for < 5% of total catch. Because fleet-specific discard mortality rates were not available in the assessment, we used the same rates as in the CC for the fixed and trawl gears, respectively.

Survey Fleets

We developed a new index of relative abundance using the Vectorized Auto-regressive Spatio-Temporal model (VAST, Thorson, 2019) which enabled the combination of various survey fleets into management-region specific indices (see Supplementary Material for full description of the modeling effort and findings). The indices were calibrated to roughly mimic the trend of individual indices used in separate assessment efforts. The index of relative abundance peaks for AK and the CC in the early 1990s, after which it declines substantially (Figure 5). The index for BC peaks in about 2004; indices for all management areas show slight increases since 2010. In addition to these indices, we introduced a nominal catch-per-unit-effort series for 1980-2009 for BC, which was standardized externally and is identical to that used in the present management framework for that region. The rationale for including this series is that BC stakeholders felt the VAST standardization did not capture the steep declines in relative abundance they are accustomed to in their region, and preferred that the OM include data that reflect the state of knowledge in that region.

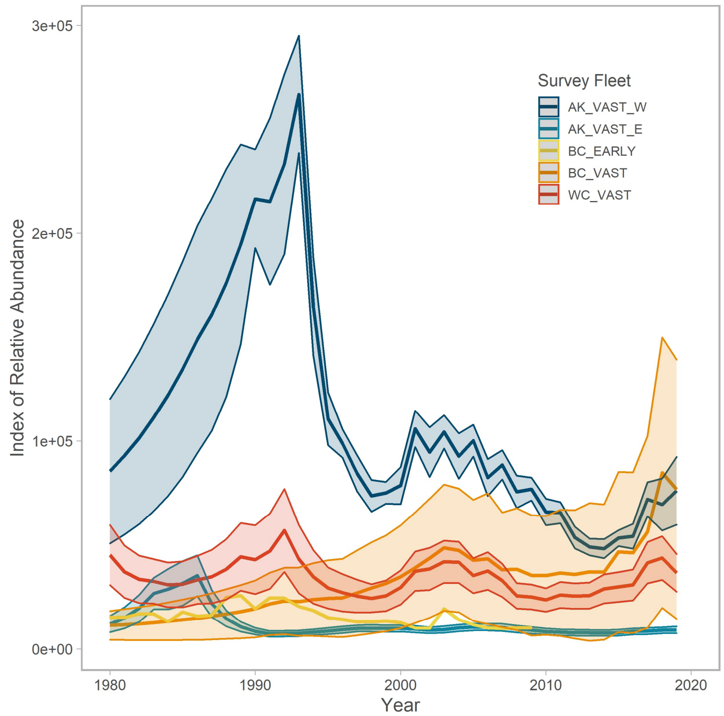

Figure 5. Indices of relative abundance developed for use in the operating model. Shaded intervals are 95% confidence intervals for estimated relative abundance in each management region. Colors correspond to the individual or collection of sub-areas surveyed by the index.

Compositional Data

Aging

We obtained compositional data from surveys and/or commercial fleets from each management region. Because we conducting a novel standardization of survey fleets using VAST to generate an index of relative abundance, there are survey fleets which are only used for compositional data (AK_GOA_LL, BC_StRS, BC_SS). Selectivity parameters are estimated for these fleets. A separate analysis was completed to determine the aging error values for each age and management region, described in a previous section.

Conditioning the Operating Model

Here, "conditioning" refers to the procedure undertaken to estimate and/or specify the parameters used in the Operating Model. As mentioned in the Demographic Parameters section, parameter values related to growth, movement and maturity were estimated externally and thus were not revisited in OM development (Table 2). The goal of this step was to find model outputs which roughly mimic reality (i.e. the general trend in spawning biomass observed from respective management regions in recent stock assessments). We do not expect model results or likelihoods to be identical to those for any current assessments, as the spatial nature of the OM, combined with the novel survey indices developed for this study, and the inclusion of movement render such comparisons unreasonable.

For this conditioning step, an estimation model (EM) that mimics the OM's structure and functional forms was developed in Template Model Builder (TMB**)**. The likelihood functions used in the maximum likelihood framework are described below, and are ordered to match the description of the OM.

Likelihood Components

This section provides an overview of the contribution to the objective function by data type. Table 2 specifies which form (e.g. normal, beta-distributed) is followed for each estimated parameter.

General parameters

The growth, fecundity, and movement parameters used in the Operating model were estimated externally, so the input parameters defining the expected distribution of length at age were fixed to those presented in Table 3. The contribution to the objective function by the priors for parameters θ following a normal distribution is:

$$L_\theta = \frac{1}{2}\left(\frac{\theta-\mu_\theta}{\sigma_\theta}\right)^2 \tag{42}$$

where μ_θ is the prior mean for the parameter; and
σ_θ is the standard deviation for the parameter prior.

The contribution to the objective function by the priors for parameters θ following a lognormal distribution is:

$$L_\theta = \frac{1}{2}\left(\frac{ln(\theta)-\mu_\theta}{\sigma_\theta}\right)^2 \tag{43}$$

Steepness

Steepness is presently fixed, but we plan to explore estimation using the following framework:

Steepness for each stock h^k is estimated using a beta-distributed penalty on stock-specific deviations from a mean h. The contribution for each stock's steepness estimate to the objective function is:

$$\begin{aligned} L_{h^k} = -\sigma_h &\left(\left[ln\left(\frac{1}{2}(h_{max}-h_{min})-h_{min}\right)+ln(0.5)\right]\right. \\ &\cdots + \ ln(h^k - h_{min} \ + 0.0001 \\ &\cdots + ln(1 - \frac{h^k - h_{min} - 0.0001}{h_{max}-h_{min}}) \end{aligned} \tag{44}$$

For clarity, we can alternatively express Equation 44 as a beta-distribution with shape and rate parameters $\tau(1-\mu), \tau\mu$:

$$-log(L_h^k) = beta(h^k, \tau(1-\mu), \tau\mu) \tag{45}$$

where $\tau = \frac{\left((h^k-h_{min})(h_{max}-h^k)\right)}{\sigma_h^2} - 1$; and

$$\mu = \frac{\left(h^k - h_{mini}\right)}{h_{max}-h_{min}}$$

Recruitment Deviations
The recruitment deviations are assumed to be lognormally distributed.

$$L_{Rec} = \frac{1}{2}\left(\sum_y \frac{\widetilde{R}_y^2}{\sigma_R^2} + b_y\, ln(\sigma_r^2)\right) \quad (46)$$

A penalty ramp like that used in Stock Synthesis may be developed for this OM. At present, b_y is set to 1 for all years.

Catch and Discards
Discards are assumed to follow a t-distribution, and accumulate on a per-fleet basis:

$$L_{Disc}^f = \frac{1}{2}(df^f + 1)ln\left[\frac{1+\left(D_{obs,y}^f - D_{pred,y}^f\right)^2}{df^f \sigma_y^{2,f}}\right] \quad (47)$$

Where df^f is the degrees of freedom for fleet *f*;
$D_{obs,y}^f$ is the observed discard for year *y* by fleet *f*;
$D_{pred,y}^f$ is the predicted discard for year *y* by fleet *f* given by the following ($Z_{y,\gamma,a}^i$ in this case is
the same as in Equation 17, considering total mortality);

$$D_{pred,y}^f = \sum_i \sum_\gamma \phi_{if} \frac{s_{\gamma,a}^f F_y^f (1-\Omega_{y,\gamma,a}^f)}{Z_{y,\gamma,a}^i} N_{y,\gamma,a}^i (1 - e^{-Z_{y,\gamma,a}^i}) \quad (48)$$

σ_y^f is the standard deviation for discards for year *y* by fleet *f*.

Annual catches follow a lognormal distribution, with a very small standard deviation (0.01) to nearly fit catches without error.

$$L_{Catch} = \frac{1}{2}\left(\sum_y \frac{\left[ln(C_{obs,y}^f) - ln(C_{pred,y}^f)\right]^2}{0.01^2}\right) \quad (49)$$

Similarly, equilibrium catch contributes to the objective function via the following; where C_{pred0}^f is fixed.

$$L_{Catc\ \_Eq} = \frac{1}{2}\left(\frac{\left[ln(C_0^f) - ln(C_{pred0}^f)\right]^2}{0.01^2}\right) \quad (50)$$

Survey Biomass
Estimates of relative abundance (biomass) from each survey are fit under the assumption that the indices for each *y* present in the survey timeseries for fleet *f*, which are equivalent across fleets, are lognormal:

$$L_{surv}^{f} = \sum_y \frac{(ln(B_{Obs,y}^{f}) - ln(q^f B_y^f))^2}{2{\epsilon_y^f}^2} + ln(\epsilon_y^f) \tag{51}$$

Recall that the adjusted standard deviation ϵ_y^f is a constant variance term plus a time-varying term calculated externally as a part of the kriging and extrapolation procedures within VAST (σ_y^f).

Compositional Data

Length composition data is not yet implemented in the model.

Age- and length-composition data from both the survey and catches are fit using the linear parameterization of the Dirichlet-multinomial compound distribution (Thorson et al., 2017). The contribution of the composition dataset from fleet *f* in year *y* to the objective function is as follows (the same approach is used for length compositions, though age is shown here):

$$L_{\pi}^{f} = \frac{\Gamma(n_y^f+1)}{\prod_a^A \Gamma(n_y^f \sum_\gamma \widetilde{\pi_{y,\gamma,a}^f}+1)} \frac{\Gamma(\theta^f n_y^f)}{\Gamma(n_y^f+\theta^f n_y^f)} \prod_a^A \frac{\Gamma(n_y^f \sum_\gamma \widetilde{\pi_{y,\gamma,a}^f}+\theta^f n_y^f \sum_\gamma \pi_{y,\gamma,a}^f)}{\Gamma(\theta^f n_y^f \sum_\gamma \pi_{y,\gamma,a}^f)} \tag{52}$$

where n_y^f is the total number of samples in the available data from fleet *f* in year *y* (for both sexes);

$\widetilde{\pi_{y,\gamma,a}}$ is the observed proportion-at-age for fleet *f* in year y of sex γ which sum to 1;

$\pi_{y,\gamma,a}^f$ is the estimated proportion-at -age for fleet *f* in year y of sex γ, which also sum to 1;

θ^f is the estimated Dirichlet-Multinomial shape parameter pertaining to the compositions (age or length) from fleet *f*. The product of θ^f and n_y^f represents the overdispersion caused by the Dirichlet distribution. (Note that there is only one shape parameter estimated per fleet dataset).

The effective sample size is given by:

$$n_{effective,y}^f = \frac{1+\theta^f n_y^f}{1+\theta^f} \tag{53}$$

Preliminary Results for OM Conditioning

The OM conditioning work is being conducted in two phases: the first, where a simplified version of the model is produced in R without estimation. Parameter values used for quantities such as R0, steepness, and selectivity are taken directly from the pre-existing assessments, where available. This step was used to roughly validate the equations presented here and act as a "sanity check" outside of the compiling requirements of Template Model Builder. The initial R exercise was able to decently fit the observed catches, and produced SSB values for each management region at the correct order of magnitude (though not with a reliable trend).

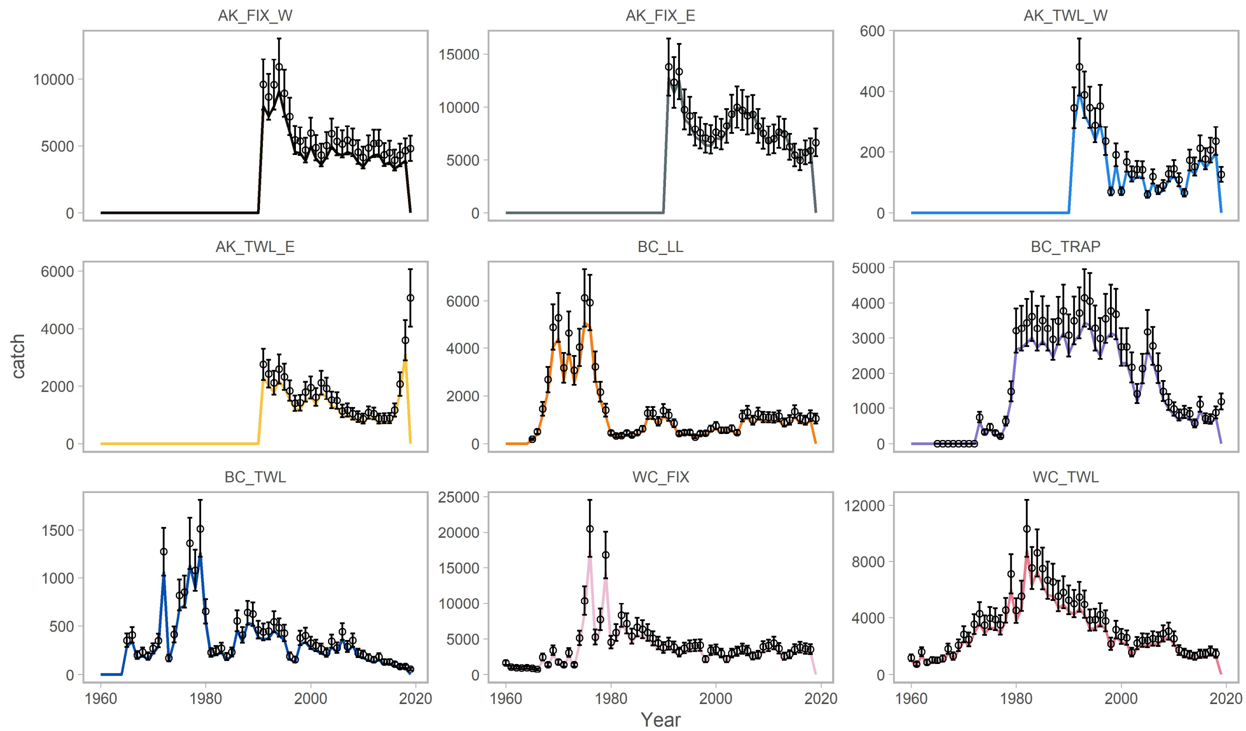

Figure 6. Catch fits using the phase 1 operating model (in R), which demonstrate the effectiveness of the hybrid tuning method. Alaskan data before 1991 are not shown at the spatial stratification for confidentiality reasons. Note: these were calculated using the final selectivity curves from the appropriate assessments, which may be more complex that the current selectivity structure in the TMB model.

Near-Term Tasks

These are the changes to the operating model the authors hope to complete by December 2020.

- Include discarding and length-composition data from various survey and fishery fleets.
- Explore more complex or alternative selectivity curves for given fleets, and time-blocked selectivity.
- Invoke estimation of *h.*
- Introduce error around (at least one of) movement parameters, or the distribution of recruits to sub-areas within stocks.
- Replace "burn-in" method of computing initial and unfished distribution with the stationary spatial distribution method (a matrix multiplication approach which does not require iterating over years).
- Enable calculation of reference points in operating model.

Tables

Region	Data Type	Description & handling notes	Reference
California Current	Landings	**2 Fleets (1990-present)**: Fixed gear (Hook & Line, and Pot), and Trawl	(Haltuch et al., 2019)
	Compositions	**Fishery-Dependent:** Ages from all fleets (via commercial port sampling) **Fishery-Independent:** Lengths & Ages from West Coast Groundfish Bottom Trawl Survey (2003-present) Ages from Triennial survey (1980-2004)	(Haltuch et al., 2019)
	Indices of Abundance	**Fishery-Dependent:** Commercial CPUE series have not been included in any recent sablefish stock assessment. **Fishery-Independent:** ●Both standardized using VAST: Triennial survey (1980-2004) and West Coast Groundfish Bottom Trawl Survey (2003-present) ⁑California Current Index of Relative Abundance (1980-2018)	(Haltuch et al., 2019)
British Columbia	Landings	**3 Fleets (1965-present):** Commercial longline trap, Longline hook, and Trawl	(Fenske et al., 2019)
	Compositions	**Fishery-Dependent:** Ages from the fishery, primarily the trap sector; lengths from commercial trawl **Fishery-Independent:** Ages from trap-based Standardized Survey (1991-2009); trap-based Stratified Random Survey (2003-present)	Fenske et al. (2019); (DFO, 2019)
	Indices of Abundance	**Fishery-Dependent**: Nominal trap fishery CPUE (1979-2009) **Fishery-Independent:** ●Standardized trap-based Standardized Survey CPUE (1991-2009); trap-based Stratified Random Survey CPUE (2003-present) ⁑British Columbia Index of Relative Abundance (1980-2018)	Fenske et al. 2019
Alaska	Landings	**2 fleets (early 1900s-present; typically cut to 1970 onward):** Fixed-gear (longline & pot) and Trawl, partitioned into East and West of 145°W.	Hanselman et al., 2019
	Compositions	**Fishery-Dependent:** Lengths (1990-present) and ages (1999-present) from Fixed-gear fishery; occasional lengths from trawl fishery **Fishery-Independent:** Ages from longline surveys (1979-present, with some collection variability)	
	Indices of Abundance	**Fishery-Dependent:** Filtered nominal CPUE scaled to area for longline fishery **Fishery-Independent:** ●Domestic Longline Survey (1979-present) and NMFS AFSC Gulf of Alaska Bottom Trawl Survey (1980-present). ⁑Gulf of Alaska Index of Relative Abundance (1980-2018) ⁑Aleutian Islands Index of Relative Abundance (1980-2018)	

Table 1. Input data available for inclusion in operating model(s). ●Original treatment of survey data (i.e. in recent stock assessments used for management). ⁑New index of relative abundance standardized using VAST, which combines survey(s) from this management region across space and time.

Symbol	Description	Operating Model Treatment	Estimated? [bounds]
Model Structure & Data			
i	sub-areas	N = 6	
k	Stocks (shared demography)	N = 4	
m	management regions m	N = 3	
f	fishing fleets	N = 9 (2 – CC, 3 – BC, 4 – AK; two fleets split into east and west)	
f	survey fleets	N = 5 (4 from VAST analysis; one from BC)	
f	fishing fleets with composition data	Ages: N = 4 (2 from WC, 1 from BC, AK_FIX split into east and west) Lengths: N = 2 (one each from AK, BC)	
f	survey fleets with composition data	Ages: N = 5 (2 – CC, 2 – BC, 1 – AK) Lengths: N = 3 (1 per management region)	
Growth*			
M^k	Stock-specific natural mortality at age. (Note equations use sub-area indexing)	Fixed at 0.2 for all ages	no
$L_{\infty,\gamma}^k$	Asymptotic length (cm)	Sex, stock and year specific (Table 3)	no
κ_γ^k	Growth rate (cm yr^{-1})	Sex, stock and year specific (Table 3)	no
σ_G	Standard deviation for length at age (cm)	Sex, stock and year specific (Table 3)	no
α^k	Coefficient of length-weight relationship (lbs/cm)	Stock specific (Table 4)	no
β^k	Allometric exponent of length-weight relationship	Stock specific (Table 4)	no
Reproduction			
ϕ_{ik}	Matrix indicating whether sub-area i is nested within stock k	See Equation 14	
h^k	steepness of the stock recruitment curve (expected proportion of R_0 at $0.2S_0$) for stock k	Estimated from beta-distribution for each stock with $h_{min} = 0.2$, $h_{max} = 1$, $h_{prior} = 0.77$ and $\sigma_h^2 = 0.117$	Presently fixed; to be estimated
E_a^k	proportion of females at age in stock k which have reached maturity at age a	See Figure 9	no
$\tilde{R}_y^k$	random annual recruitment deviations specific to stock k	normally distributed with mean zero and standard deviation σ_R	yes
σ_R	Standard deviation of recruitment deviations	log(1.4)	no
R_0^k	Unfished recruitment by stock	Start values based roughly on assessments	yes
Catches			
ϕ_{if}	Matrix indicating whether fleet f occurs in sub-area i		
$\beta_{1,2,3,4}^{y,f,\gamma,a/l}$	Age @ inflection point, slope @ inflection point, asymptotic selection and male offset for logistic retention curve	Fleet-specific (mirrored across management areas for like gear types)	Not yet implemented
$w_a^{k,f}$	stock- and fleet-specific weight-at-age of captured fish	Defined by parameters in Table 4	no
a_{50}, δ	Fishery selectivity parameters	Fleet-specific, see Table 6	yes
Surveys			
σ_S	Annual standard deviation of relative abundance by fleet	See Table 7	no (estimated externally)

$w_a^{k,f}$	stock- and fleet-specific weight-at-age of sampled fish	Defined by parameters in Table 4	
a_{50}, δ	Survey selectivity parameters placeholder	Fleet-specific, see Figure 10	
q^f	Survey catchability coefficient		yes
Age & Length Compositions			
π^f	Fleet- specific proportion at length or age; can be year- and/or gender-specific		
θ_a^f	Fleet-specific Dirichlet-Multinominal parameter		yes
σ_A	Standard deviation at age in aging error matrix	Management region, age, and sex-specific	no (estimated externally)
Movement			
$\boldsymbol{X}_a^{i,j}$	Sub-area age-based movement matrix	See Figure 8	no (estimated externally)
τ_{yik}	Proportional distribution of recruits in year *y* spawned in stock *k* to sub-area *i; i ε k*	See Table 5	no
ϵ_τ	Lognormally-distributed variance term for τ_{yik}	$\epsilon_\tau \sim lognormal(0,0.025)$	Not yet implemented

Table 2. Overview of parameters used in operating model. Acronyms referring to OM-specific regions are explained in-text and depicted in Figure 1. *In text, growth equations use sub-area indexing (*i*) to clarify the expected length-at-age as fish move among sub-areas from different stocks. In practice, sub-areas belonging to the same stock *k* share growth patterns.

Stock (sub-areas)	Sex	Period	L_{∞}^{k} (cm)	κ_{γ}^{k} (cm yr^{-1})	σ^{k} (cm)
R1 (C1)	Fem	<2010	60.3	0.29	4.98
R1 (C1)	Fem	2010+	62.75	0.16	4.8
R1 (C1)	Male	All years	55.07	0.28	3.89
R2 (C2 & B1)	Fem	<2010	69.36	0.22	11.8
R2 (C2 & B1)	Fem	2010+	68.08	0.18	9.68
R2 (C2 & B1)	Male	All years	59.02	0.21	7.16
R3 (B2 & A2)	Fem	<2010	73.51	0.76	8.43
R3 (B2 & A2)	Fem	2010+	75.04	0.35	10.8
R3 (B2 & A2)	Male	All years	65.66	0.34	9.9
R4 (A1)	Fem	All years	81.43	0.14	6.57
R4 (A1)	Male	All years	68.34	0.2	4.75

Table 3. Growth parameters used in Operating Model, updated following methods from Kapur et al. 2020.

Stock (sub-areas)	α^{k}	β^{k}
R1 (C1)	8.58e-6	3.50
R2 (C2 & B1)	9.22e-6	3.28
R3 (B2 & A2)	5.95e-6	3.16
R4 (A1)	3.32e-6	3.27

Table 4. Weight-at-length parameters for each stock. These follow the syntax $W(L) = \alpha^{k} L^{\beta^{k}}$. For stock R1 these were taken from the appropriate regional stock assessment. For blended stock R2 we calculated the average from the two contributing management regions; and for blended stocks R3 and R4 we took the average of the BC assessment values, and those values inflated by 15% (the most recent AK assessment uses weight-at-age).

	Sub-area					
Stock	**A1**	**A2**	**B2**	**B1**	**C2**	**C1**
R1	0	0	0	0	0	1
R2	0	0	0	0.25	0.75	0
R3	0	0.75	0.25	0	0	0
R4	1	0	0	0	0	0

Table 5. Proportional distribution of recruits in year y spawned in stock k to sub-area i; $i \varepsilon k$. These values are represented in equations with symbol τ_{yik}, and are roughly based on the proportional area of each sub-area within each stock. Future iterations of the model may estimate or perform sensitivities to these values.

Mgmt. Region	Fleet Name	Fleet Type(s)	Years	Selectivity form, if applicable	Note
AK	AK_FIX_W	Comm. catches, age & length comps	1977-2019	Age-based logistic	Length comps not presently in model
AK	AK_FIX_E	Comm. catches, age & length comps	1977-2019	Age-based logistic	Length comps not presently in model
AK	AK_TWL_W	Comm. catches, age & length comps	1970-2019	Age-based logistic	Length comps not presently in model
AK	AK_TWL_E	Comm. catches, age & length comps	1970-2019	Age-based logistic	Length comps not presently in model
AK	AK_VAST_W	Standardized index of relative abundance	1980-2019	Age-based logistic	
AK	AK_VAST_E	Standardized index of relative abundance	1980-2019	Age-based logistic	
AK	AK_LLSURV_W	Length comps			Not presently in model
AK	AK_LLSURV_E	Length comps			Not presently in model
AK	AK_GOA_SURV	Age comps	1984-2019 (sparse)	Age-based logistic	
BC	BC_EARLY	Standardized index of relative abundance	1979-2009		Separate from VAST
BC	BC_LL	Comm. catches, length comps, discards	1965-2019	Length-based dome	Discards, length comps not presently in model
BC	BC_TRAP	Comm. catches, age & length comps	1973-2019, some age comps from 1965	Length-based dome	Length comps not presently in model
BC	BC_TWL	Comm. catches, length comps, discards	1965-2019	Length-based dome	Discards, length comps not presently in model
BC	BC_VAST	Standardized index of relative abundance	1980-2019	Age-based logistic	
BC	BC_StRS	Age comps	2003-2018 (intermittent)	Length-based logistic	
BC	BC_SS	Age Comps	1990-2009	Length-based logistic	
WC	WC_NWCBO	Length comps (conditional age at length)			not presently in model

WC	WC_FIX	Comm. catches, age comps, discards	1960-2019	Age-based dome	Discards not presently in model
WC	WC_TWL	Comm. catches, age comps, discards	1960-2019	Age-based dome	Discards not presently in model
WC	WC_VAST	Standardized index of relative abundance	1980-2019	Age-based logistic	

Table 6. Survey and fishery fleets, year range of available data, and form of selectivity curve used in the operating model

Year	AK_VAST_W	AK_VAST_E	BC_EARLY	BC_VAST	WC_VAST
1980	85490.42	11844.21	15312	11309.92	45139.33
1981	92941.84	14891.19	15056	11678.77	37117.17
1982	101471.5	19511.17	16973	12103.36	33463.59
1983	111260.4	26579.37	16819	12583.48	32536.62
1984	122139.8	28712.49	13059	13240.39	30753.12
1985	134583.8	31518.67	17687	13951.19	31046.98
1986	148841.8	35213.26	15602	14719.76	33176.37
1987	160843.7	21777.45	16160	15688.89	34757.98
1988	176137.1	14603.89	24736	16730.72	38317.32
1989	194668.3	10685.82	25695	17851.26	44310.35
1990	216620.8	8483.12	19222	19213.21	42899.32
1991	215300.5	7135.229	24600	21590.05	47032.27
1992	233334.5	7266.306	24363	23014.18	56962.95
1993	266910.2	7469.622	20380	22708.89	43239.75
1994	164621.8	8036.086	18397	23715.55	34665.6
1995	110687.2	8509.1	15020	24255.34	29311.97
1996	98835.34	9223.849	14087	24553.75	27053.85
1997	84535.73	9820.933	12956	26905.06	25401.92
1998	73640.18	9961.155	13020	29330.42	24313.43
1999	75055.34	9753.784	13426	31476.9	25538.63
2000	78620.82	9848.973	12667	34741.09	29557.56
2001	105895.9	9496.339	10082	39082.15	37336.72
2002	94603.57	8925.449	9899	43580.75	38467.54
2003	104395.4	9333.535	19222	48674.41	41921.01
2004	92731.59	10050.26	14009	47339.26	41640.28
2005	100160.7	10510.59	11615	42589.82	35210.53
2006	82371.57	10425.91	10034	43392.39	37454.85
2007	88581.95	10255	9705	37987.73	32958.16
2008	75565.75	9583.784	10042	38326.67	25540.72
2009	76902.69	8962.89	10090	35376.49	24987.56
2010	65872.5	8307.914		35320.64	23534.76
2011	65537.13	7974.877		36582.28	25957.33
2012	53570.87	7783.33		35818.54	25442.46
2013	49030.48	7725.194		37050.9	25704.96
2014	48220.06	7556.013		36947.79	28852.67
2015	53276.34	7782.333		46601.47	29827.79
2016	54215.37	8397.037		46211.45	30761.56
2017	71956.33	8610.917		56213.88	41434.33
2018	69442.7	8935.047		84801.93	43681.04
2019	76100.89	9129.285		76737.42	36580.18

Table 7. Input standardized survey biomass, by management region. These values were obtained via standardization using VAST, except for the Nominal Trap CPUE from BC (BC_EARLY), which is an index of relative abundance calculated externally.

Year	AK_VAST_W	AK_VAST_E	BC_EARLY	BC_VAST	WC_VAST
1980	0.406752	0.328276	317	0.611938	0.319574
1981	0.408851	0.340564	317	0.635889	0.338179
1982	0.408056	0.332743	317	0.656721	0.341078
1983	0.403841	0.285848	317	0.67473	0.329227
1984	0.397235	0.33989	317	0.687555	0.351349
1985	0.385498	0.33947	317	0.698076	0.357379
1986	0.36701	0.278307	317	0.706383	0.347328
1987	0.347922	0.297317	317	0.711233	0.371103
1988	0.310889	0.265181	317	0.714275	0.379249
1989	0.246523	0.21905	317	0.715527	0.368043
1990	0.109554	0.186984	317	0.709769	0.3856
1991	0.186128	0.198053	317	0.695205	0.385794
1992	0.185434	0.192326	317	0.693551	0.350337
1993	0.105632	0.186314	317	0.727208	0.379027
1994	0.143095	0.17393	317	0.740594	0.369735
1995	0.115341	0.180291	317	0.757405	0.337557
1996	0.06909	0.171972	317	0.76708	0.331382
1997	0.102603	0.157742	317	0.757796	0.310242
1998	0.103327	0.160622	317	0.746741	0.276165
1999	0.069259	0.1657	317	0.73424	0.28652
2000	0.113971	0.155528	317	0.713023	0.278836
2001	0.082158	0.166487	317	0.68205	0.253562
2002	0.126643	0.166292	317	0.65791	0.264514
2003	0.076738	0.169843	317	0.624209	0.241392
2004	0.117855	0.154768	317	0.628021	0.238613
2005	0.076346	0.161502	317	0.68413	0.240182
2006	0.10797	0.151621	317	0.725849	0.240872
2007	0.078785	0.168784	317	0.726562	0.241056
2008	0.103214	0.172653	317	0.764443	0.241302
2009	0.071735	0.178863	317	0.819658	0.241091
2010	0.097777	0.176743	317	0.810336	0.240457
2011	0.077381	0.185168	317	0.830689	0.240241
2012	0.098034	0.181677	317	0.860539	0.240469
2013	0.080957	0.18747	317	0.894461	0.242346
2014	0.095251	0.185527	317	0.88125	0.240511
2015	0.072554	0.187814	317	0.828426	0.240428
2016	0.109844	0.17082	317	0.836688	0.24037
2017	0.113449	0.175619	317	0.81985	0.240414
2018	0.180082	0.163328	317	0.76757	0.240674

2019	0.214749	0.184622	317	0.812192	0.247233

Table 8. Input standardized survey log standard deviations (σ_S^2), by management region. These were standardized using VAST, except for the Nominal Trap CPUE from BC (BC_EARLY), which is an index of relative abundance calculated externally.

Additional Figures

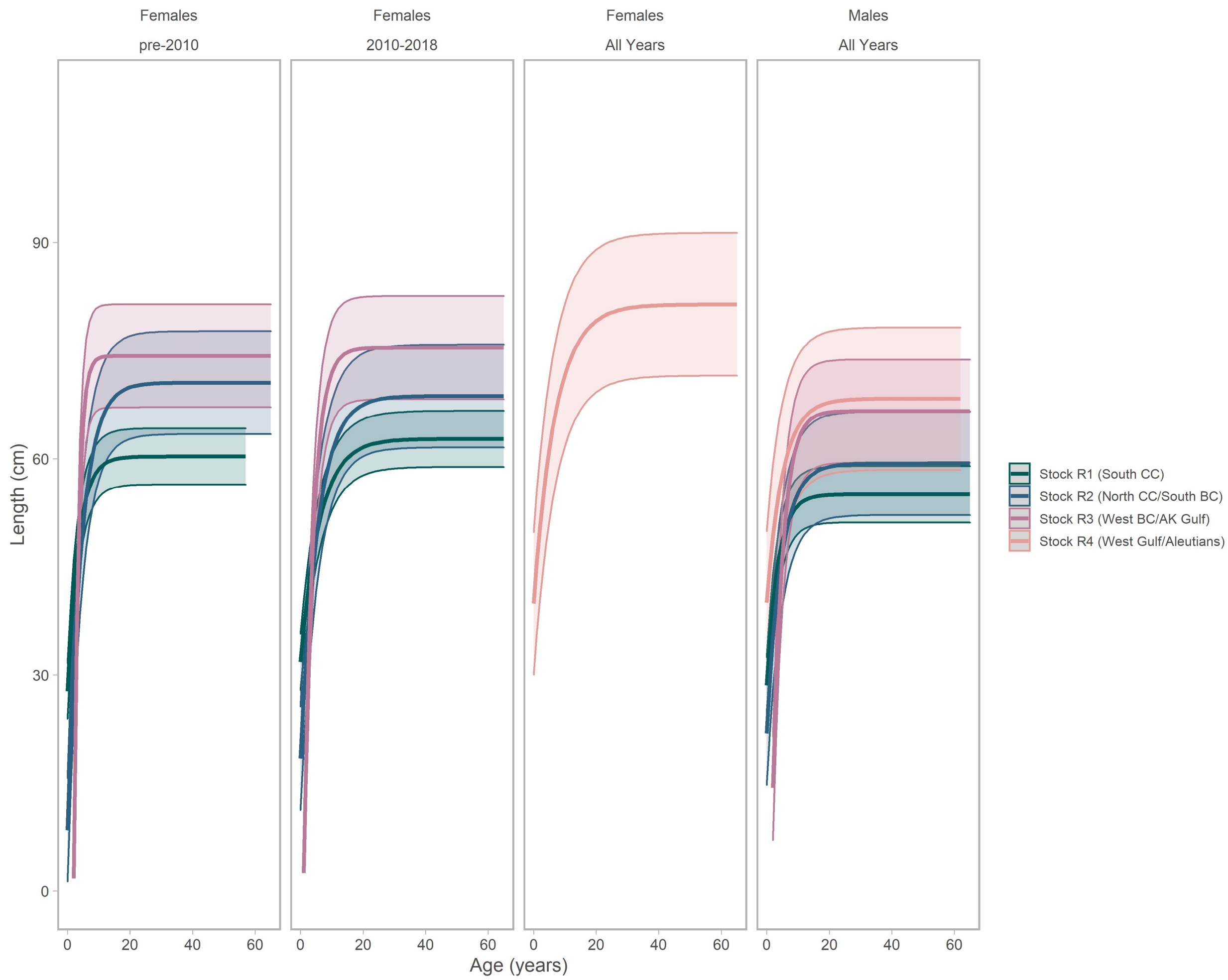

Figure 7. Input growth curves by stock. Stock labels correspond to light grey boxes in Figure 1.

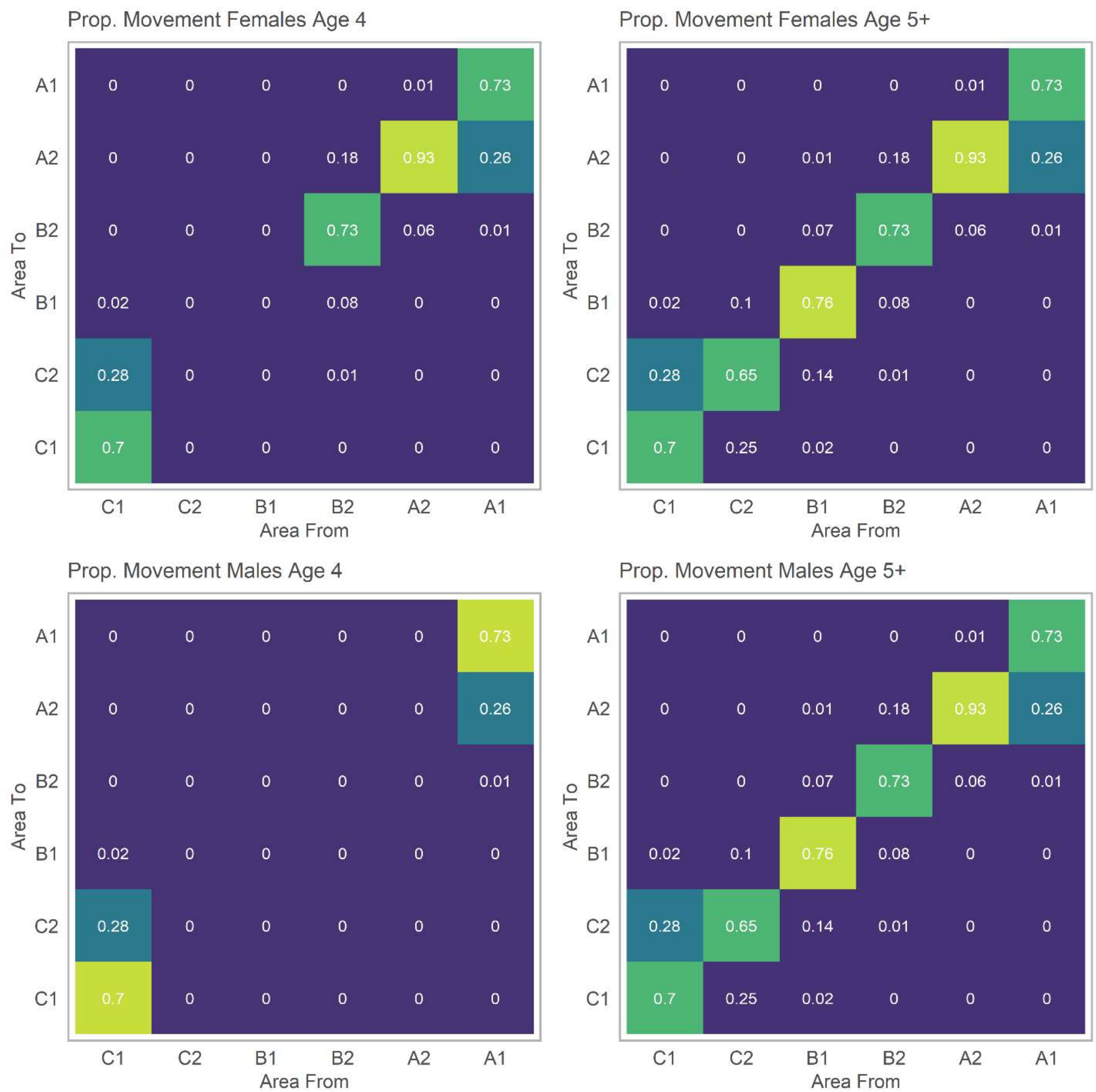

Figure 8. Fixed input movement rates by sex and age. Both males and females below age four do not move among subareas.

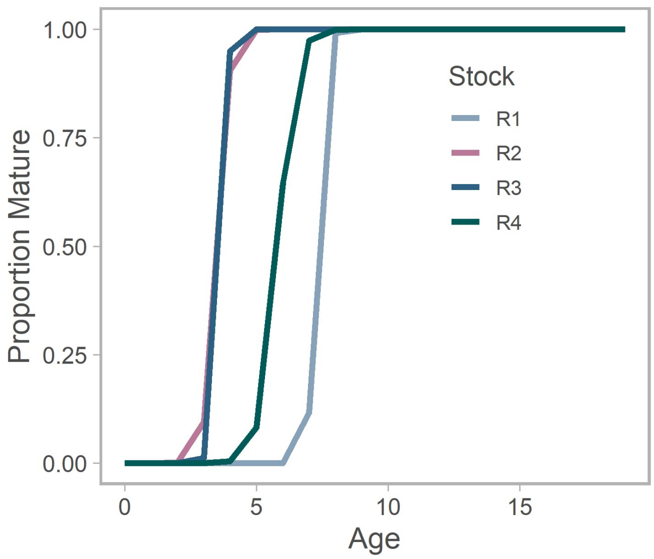

Figure 9. Fixed input maturity-at-age for female sablefish. Values are only shown through age 20 for clarity. Ages at 50% and 75% maturity were estimated for a logistic curve based on macroscopic data from all three regions in an analysis external to this work (Williams et al., in prep).

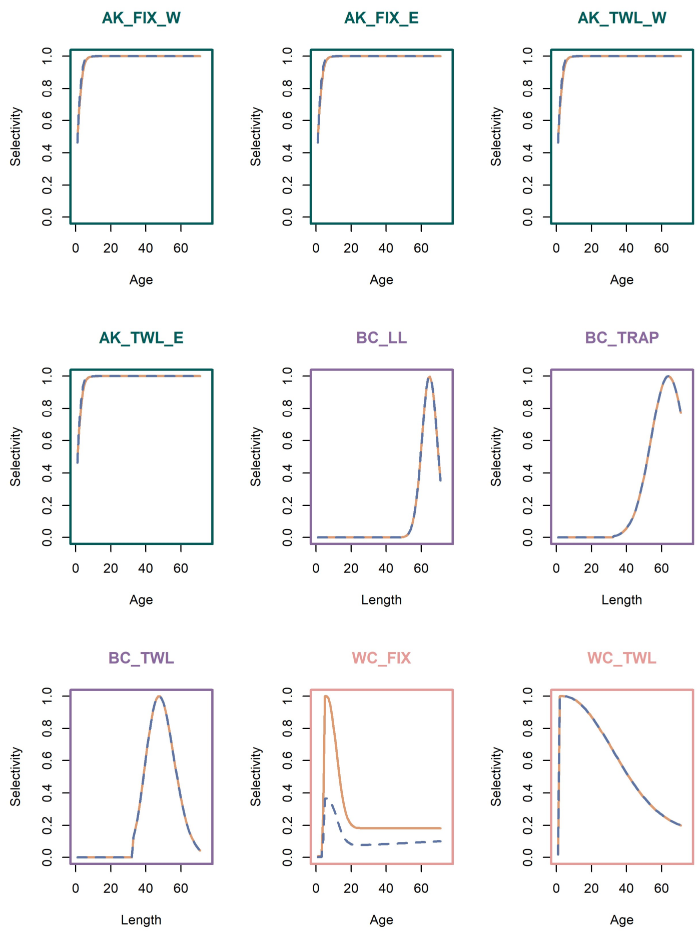

Figure 10. Selectivity curves used in pre-existing assessments for commercial fishery fleets, by age or length and sex (females = solid lines, males = dashed lines). The shape and starting parameters in the OM are based on these values, with the change that more complex shapes (i.e. double-normal with dog legs) are substituted by simple normal or gamma distributions.

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