AN APPROACH FOR COMPUTING E_{MSY}, B_{MSY} and MSY FOR THE CSNA

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A. Introduction

The OFL (Overfishing Limit) for the Central Subpopulation of Northern Anchovy (CSNA) is currently set to an estimate of long-term Maximum Sustainable Yield (MSY), with the Acceptable Biological Catch (ABC) established as 25% of the OFL. A "near-term" value for the OFL could be calculated by multiplying an estimate of current biomass (either spawning biomass, e.g. from Daily Egg Production Method or 1+ biomass, e.g. from the acoustic-trawl method) by the exploitation rate corresponding to MSY, E_{MSY}[1]). Thus, the steps in calculating an OFL involve estimating current biomass and multiplying it by a value for E_{MSY}. This document provides an approach for computing E_{MSY}.

This document first outlines, and then applies, a technique to analyse the existing information on spawning biomass and age-0 abundance to characterize a relationship between these variables (Section B). This relationship is critical to Section C that then uses the estimated stock-recruitment relationship (and associated uncertainty), along with other biological and fishery parameters, to calculate probability distributions for future catch, and hence E_{MSY}.

B. Fitting the stock-recruitment relationship

Basic structure

The data available are estimates of recruitment (age-0 abundance) and spawning biomass from the most recent age-structured assessment (Jacobson *et al.* 1995; Table 1 (Table 1). These data are modelled using one of four models: (a) Beverton-Holt with auto-correlation, (b) Beverton-Holt no auto-correlation, (c) Ricker with auto-correlation, and (d) Ricker no auto-correlation. The likelihood function is the same for all four models, i.e.:

$$-\ell nL = n\ell n\sigma_R + \tfrac{1}{2\sigma_R^2}\left(\varepsilon_1^2 + \sum_{y=2}^{n}\frac{(\varepsilon_y - \rho\varepsilon_{y-1})^2}{1-\rho^2}\right) \tag{1}$$

where ε_y is the residual for year y:

$$\varepsilon_y = \ell\text{n}R_y^{obs} - \ell\text{n}\hat{R}_y \tag{2}$$

R_y^{obs} is the observed age-0 abundance for year y (Table 1), $\hat{R}_y$ is the model-estimate of the age-0 abundance for year y, ρ is the extent of auto-correlation in the deviations about the stock-recruitment relationship, σ_R quantifies the extent of variation about the stock-recruitment relationship, and n is the number of data points (32).

The model-predicted recruitment depends on the assumed stock-recruitment relationship, i.e.:

[1] Often referred to as F_{MSY} at the PFMC, although "F" usually refers to an instantaneous rate. Thus, F in Equations 6 and 7 are not E_{MSY}.

$$\hat{R}_y = \begin{cases} \frac{4hR_0(SSB_y/SSB_0)}{(1-h)+(5h-1)SSB_y/SSB_0} & \text{Beverton-Holt} \\ R_0(SSB_y/SSB_0)e^{-\ell n(5h)(SSB_y/SSB_0-1)/0.8} & \text{Ricker} \end{cases} \tag{3}$$

where R_0 is the expected unfished recruitment, SSB_y is the spawning biomass in year y, SSB_0 is the unfished spawning biomass (= R_0 $SSBR_0$), $SSBR_0$ is the spawning biomass-per-recruit in the absence of fishing, and h is the steepness of the stock-recruitment relationship. The value of $SSBR_0$ is computed as:

$$SSBR_0 = \sum_a \tilde{N}_a P_a w_a^p \tag{4}$$

where $\tilde{N}_a$ is the numbers-per-recruit:

$$\tilde{N}_a = \begin{cases} 1 & \text{if } a = 0 \\ \tilde{N}_{a-1} e^{-M} & \text{if } 1 \le a < x \\ \tilde{N}_{x-1} e^{-M} / (1-e^{-M}) & \text{if } a = x \end{cases} \tag{5}$$

M is the rate of natural mortality, P_a is the proportion mature at age, w_a^p is the population weight-at-age, and x is the plus-group age (age 6). The values for the biological parameters are listed in Table 2.

Implementation
Posterior distributions are constructed for the four cases using the Sample-Importance-Resample (SIR) algorithm. The prior distributions[2] are:

Parameter	Beverton-Holt	Ricker
Steepness, h	U[0.2, 1]	U[0.2, 3]
Unfished recruitment, $\ell n R_0$	U[ln(5000), ln(200,000)]	U[ln(5000), ln(200,000)]
Auto-correlation, ρ [1]	[-0.99, 0.99]	[-0.99, 0.99]
Recruitment standard deviation, σ_R	U[0, 2]	U[0, 2]

1: if not set to zero

A total of 10,000,000 samples are drawn from the priors to construct a posterior sample of 1,000. The performance of the SIR algorithm is evaluated by the number of unique parameter vectors in the posterior sample. In addition, weights for the four cases can be constructed using the Bayes factor.

[2] It would be desirable to examine how sensitive the results are to alternative prior specifications, and possibly consider more informative priors and/or alternative formulations of the priors if sensitivity is high. In particular the steepness prior might be informed by consideration of other CPS stocks. The priors for steepness and the R_0 might be informed by life history considerations, such as the approach of Mangel *et al*. (2010), and the autocorrelation prior might be informed by consideration of the degree of autocorrelation in plausible environmental drivers of recruitment. However, the information to conduct such additional analysis is not currently available.

Results
Table 3 summarizes the results of applying the SIR algorithm. The number of unique samples suggests that the algorithm has converged. Figures 1-4 show diagnostic plots for the four Bayesian analyses. Note that the data support an estimate of steepness near the lower bound of 0.2, but some of these samples will be excluded in the analyses for determining E_{MSY} owing to the "evolutionary exclusion" criterion (see below). Table 3 lists the marginal likelihoods, which could be used to compare models.

C. Computing a yield function

Methods
The yield and spawning biomass values used to construct the yield function are computed by projecting an age-structured model forward. The basic dynamics of this model are:

$$N_{y+1,a} = \begin{cases} R_{y+1} & \text{if } a = 0 \\ N_{y,a-1} e^{-(M + S_{a-1} F_y)} & \text{if } 1 \le a < x \\ N_{y,x-1} e^{-(M + S_{x-1} F_y)} + N_{y,x} e^{-(M + S_x F_y)} & \text{if } a = x \end{cases} \qquad (6)$$

where $N_{y,a}$ is the number of animals of age a at the start of year y, F_y is the fully-selected fishing mortality during year y, S_a is fishery selectivity for an animal of age a, and R_y is the generated age-0 abundance for year y (accounting for the log-normal bias-correction factor). The catch during year y is computed using the equation:

$$C_y = \sum_a w_a^c \tfrac{S_a F_y}{M + S_a F_y} N_{y,a} (1 - e^{-(M + S_a F_y)}) \qquad (7)$$

where w_a^c is the population weight-at-age (Table 2).

The value of F_y is computed by solving Equation 7 where the catch is given by:

$$TAC_y = E_{\text{targ}}\, B_y\, e^{\eta_y - \sigma_I^2/2} \qquad \eta_y \sim N(0; \sigma_I^2) \qquad (8)$$

where E_{targ} is the exploitation rate (aka "E_{MSY}"), B_y is the biomass to which E_{MSY} applies (either the 1+ biomass or spawning biomass), and σ_I is the extent of observation error. Note that this approach accounts for the fact that the selectivity of the approach used to determine biomass differs from that of the fishery. Note also that no account is taken of implementation error (unless σ_I is assumed to be include both observation and implementation error). The value of σ_I is set to 0.4 for both spawning biomass and 1+ biomass (Dorval *et al*., 2018; Zwolinski *et al*., 2017).

Implementation
A total of 10,000 projections are conducted for each choice for E_{targ} (10 replications for each of the 1,000 samples from the posterior). Results for simulations in which the population is not sustainable (> 1% of unfished spawning biomass) when $E_{\text{targ}} = 0$ are ignored (as such cases should be "evolutionarily excluded"). The yield curves and reference points are based on the values for catch and population size after 500 years (more than sufficient for transient effects to be eliminated).

The results are (see Figs 5-8 for example) are summarized by:

a) The relationships between E_{targ} (exploitation rate) and both median and expected (mean) yield (over replicates) when the value of R_0=1, along with 5%, 10% 25%, 75%, 90% and 95% simulation intervals (upper left panel). The exploitation rate at which the median or expected yield is maximized is a candidate definition of E_{MSY}.

b) The relationships between depletion (i.e., spawning biomass as a fraction of unfished biomass) and both median and expected yield when the value of R_0=1 (upper right panel). The relative spawning biomasses at which MSY occurs is a candidate definition for SSB_{MSY}/SSB_{B0}.
c) The relationships between E_{targ} and both median and expected yield, along with 5%, 10% 25%, 75%, 90% and 95% simulation intervals (lower left panel) when the value of R_0 for each simulation is set to the estimate from the stock-recruitment analysis. The exploitation rate at which the median or expected yield is maximized is a candidate definition of E_{MSY}.
d) The relationships between spawning biomass and both median and expected yield when the value of R_0 for each simulation is set to the estimate from the stock-recruitment analysis (lower right panel). The peaks of the yield curves are estimates of MSY and the spawning biomass at which MSY occurs is a definition for SSB_{MSY}.

Results are shown for the four stock-recruitment relationship assumptions in Section A, along with two sensitivity scenarios: (a) restricting the prior for the extent of auto-correlation to positive autocorrelation only, and (b) setting natural mortality to 1.1yr^{-1} (U(-0.99, 0.99) prior only). The results or the second sensitivity scenario should be interpreted with caution because the spawning biomass and age-0 abundance on which the stock-recruitment relationship is based on assuming that M=0.8yr^{-1}. This sensitivity scenario should ideally be conducted by rerunning the Jacobsen *et al*. 1995) stock assessment while setting M=1.1yr^{-1}.

Results

The key model outputs are listed in Tables 4 and 5. Results are shown when current biomass is expressed as spawning biomass and as 1+ biomass. The proportion of samples from the posteriors that are not excluded due to the "evolutionary constraint" is greater for the Beverton-Holt stock-recruitment relationship than for the Ricker stock-recruitment relationship and for the lower value for M (almost 20% of the samples from the posterior are excluded for the Ricker stock-recruitment relationship with high natural mortality; Table 5). The former result reflects that the Beverton-Holt results reflect a more productive stock than the Ricker results in general.

Given the current decision in front of the Council, the focus for discussion is on E_{MSY}. The values for E_{MSY} are sensitive to all of the factors explored (not unexpectedly): with ranges of 0.13-0.86 for spawning biomass; 0.11-0.60 for 1+ biomass (although the upper values are based on M=1.1yr^{-1} and should be interpreted with caution). The value for E_{MSY} is larger when biomass is expressed as spawning biomass than when it is expressed as 1+ biomass (because spawning biomass is smaller than 1+ biomass) and it is larger when the stock-recruitment relationship is Beverton-Holt rather than Ricker. Considering the mean rather than median estimates also leads to higher values for E_{MSY}. Allowing for autocorrelation in recruitment (particularly positive autocorrelation) leads to higher values for E_{MSY}.

The values for SSB_{MSY}/SSB_0 are less sensitive to model assumptions than the values for E_{MSY} and range from 0.28 to 0.46 (biomass expressed as spawning biomass) and 0.28 to 0.42 (biomass expressed as 1+ biomass). The major factor determining the value of SSB_{MSY}/SSB_0 is the form of the stock-recruitment relationship. Values for MSY are provided in Tables 4 and 5, but given the extreme variation in recruitment of anchovy, catches will seldom be close to MSY (see the lower left panels of Figures 5-8).

Acknowledgements
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Table 1. The spawning biomass and age-0 abundance data used in the analyses (source: Jacobson *et al.*, 1995)

Year	**SSB (10^3 mt)**	**Age-0 (10^6 individuals)**
1963	612	12769
1964	356	15923
1965	236	35692
1966	230	19077
1967	206	18538
1968	173	63846
1969	198	19846
1970	172	63846
1971	138	19846
1972	383	334462
1973	474	115000
1974	932	204000
1975	1069	76154
1976	901	38154
1977	520	50308
1978	337	136154
1979	654	73923
1980	490	48538
1981	320	157692
1982	711	17077
1983	395	148846
1984	555	85769
1985	715	23385
1986	409	16692
1987	227	98846
1988	167	14308
1989	239	10462
1990	152	20769
1991	171	13385
1992	145	18077
1993	154	72154
1994	388	43154

Table 2 Biological parameters for the CSNA (source: Anon, 2016)

Age	**M (yr^{-1})**	S_a	P_a	W_a^c **(kg)**	W_a^p **(kg)**
0	0.8	0.161	0	0.0130	0.0000
1	0.8	0.666	0.55	0.0165	0.0096
2	0.8	0.993	1	0.0196	0.0150
3	0.8	1.000	1	0.0221	0.0190
4	0.8	0.668	1	0.0253	0.0217
5	0.8	0.300	1	0.0284	0.0243
6	0.8	0.000	1	0.0311	0.0311

Table 3. Results of the application of the SIR algorithm.

Model	**Marginal Likelihood**	**Unique samples**
Beverton-Holt with auto-correlation	84878.78	992
Beverton-Holt no auto-correlation	464459.50	996
Ricker with auto-correlation	12824.38	974
Ricker no auto-correlation	70277.15	986

Table 4. Summary statistics for the stochastic projections

(a) OFL based on estimates of spawning stock biomass

Metric	Beverton-Holt			Ricker		
Autocorrelation, ρ	U(-0.99, 0.99)	0	U(0, 0.99)	U(-0.99, 0.99)	0	U(0, 0.99)
Proportion accepted	0.94	0.91	0.94	0.85	0.85	0.90
Relative R_0						
E_{MSY} (medians)	0.30	0.28	0.33	0.15	0.13	0.18
E_{MSY} (means)	0.55	0.50	0.59	0.26	0.22	0.27
SSB_{MSY}/SSB_0 (medians)	0.26	0.23	0.27	0.19	0.19	0.23
SSB_{MSY}/SSB_0 (means)	0.22	0.22	0.23	0.23	0.24	0.26
Absolute R_0						
E_{MSY} (medians)	0.32	0.28	0.35	0.14	0.14	0.16
E_{MSY} (means)	0.51	0.45	0.51	0.26	0.22	0.27
SSB_{MSY} (medians) (‘000t)	103	98	105	108	96	137
SSB_{MSY} (means) (‘000t)	104	102	111	123	134	141
SSB_0 (medians) (‘000t)	289	275	288	267	273	301
SSB_0 (means) (‘000t)	364	349	357	339	354	372
MSY (medians) (‘000t)	32	29	36	19	18	23
MSY (means) (‘000t)	56	49	60	37	35	42
SSB_{MSY}/SSB_0 (medians)	0.36	0.36	0.36	0.41	0.35	0.46
SSB_{MSY}/SSB_0 (means)	0.28	0.29	0.31	0.36	0.38	0.38

(b) OFL based on estimates of 1+ biomass

Metric	Beverton-Holt			Ricker		
Autocorrelation, ρ	U(-0.99, 0.99)	0	U(0, 0.99)	U(-0.99, 0.99)	0	U(0, 0.99)
Proportion accepted	0.94	0.91	0.94	0.85	0.85	0.90
Relative R_0						
E_{MSY} (medians)	0.25	0.23	0.29	0.13	0.11	0.14
E_{MSY} (means)	0.42	0.39	0.45	0.21	0.18	0.22
SSB_{MSY}/SSB_0 (medians)	0.25	0.23	0.24	0.18	0.19	0.25
SSB_{MSY}/SSB_0 (means)	0.22	0.22	0.23	0.22	0.23	0.26
Absolute R_0						
E_{MSY} (medians)	0.24	0.22	0.28	0.12	0.11	0.14
E_{MSY} (means)	0.39	0.35	0.40	0.21	0.18	0.22
SSB_{MSY} (medians) (‘000t)	111	101	103	102	102	128
SSB_{MSY} (means) (‘000t)	104	101	108	122	132	138
SSB_0 (medians) (‘000t)	289	275	288	267	273	301
SSB_0 (means) (‘000t)	364	349	357	339	354	372
MSY (medians) (‘000t)	32	29	35	20	18	23
MSY (means) (‘000t)	57	50	61	38	35	43
SSB_{MSY}/SSB_0 (medians)	0.38	0.37	0.36	0.38	0.37	0.42
SSB_{MSY}/SSB_0 (means)	0.28	0.29	0.30	0.36	0.37	0.37

Table 5. Sensitivity of the results of U(-0.99, 0.99) projections to the value of natural mortality.

	Beverton-Holt stock-recruitment relationship				Ricker stock-recruitment relationship			
	SSB		1+		SSB		1+	
Autocorrelation, ρ	M=0.8yr^{-1}	M=1.1yr^{-1}	M=0.8yr^{-1}	M=1.1yr^{-1}	M=0.8yr^{-1}	M=1.1yr^{-1}	M=0.8yr^{-1}	M=1.1yr^{-1}
Proportion accepted	0.94	0.91	0.94	0.91	0.85	0.81	0.85	0.81
Relative R_0								
E_{MSY} (medians)	0.30	0.46	0.25	0.35	0.15	0.21	0.13	0.15
E_{MSY} (means)	0.55	0.86	0.42	0.60	0.26	0.41	0.21	0.30
SSB_{MSY}/SSB_0 (medians)	0.26	0.21	0.25	0.20	0.19	0.13	0.18	0.15
SSB_{MSY}/SSB_0 (means)	0.22	0.21	0.22	0.21	0.23	0.19	0.22	0.20
Absolute R_0								
E_{MSY} (medians)	0.32	0.45	0.24	0.33	0.14	0.21	0.12	0.16
E_{MSY} (means)	0.51	0.79	0.39	0.55	0.26	0.40	0.21	0.29
SSB_{MSY} (medians) (‘000t)	103	47	111	48	108	34	102	34
SSB_{MSY} (means) (‘000t)	104	50	104	50	123	55	122	56
SSB_0 (medians) (‘000t)	289	132	289	132	267	109	267	109
SSB_0 (means) (‘000t)	364	174	364	174	339	155	339	155
MSY (medians) (‘000t)	32	22	32	22	19	12	20	12
MSY (means) (‘000t)	56	43	57	44	37	27	38	28
SSB_{MSY}/SSB_0 (medians)	0.36	0.36	0.38	0.36	0.41	0.31	0.38	0.31
SSB_{MSY}/SSB_0 (means)	0.28	0.29	0.28	0.29	0.36	0.35	0.36	0.36

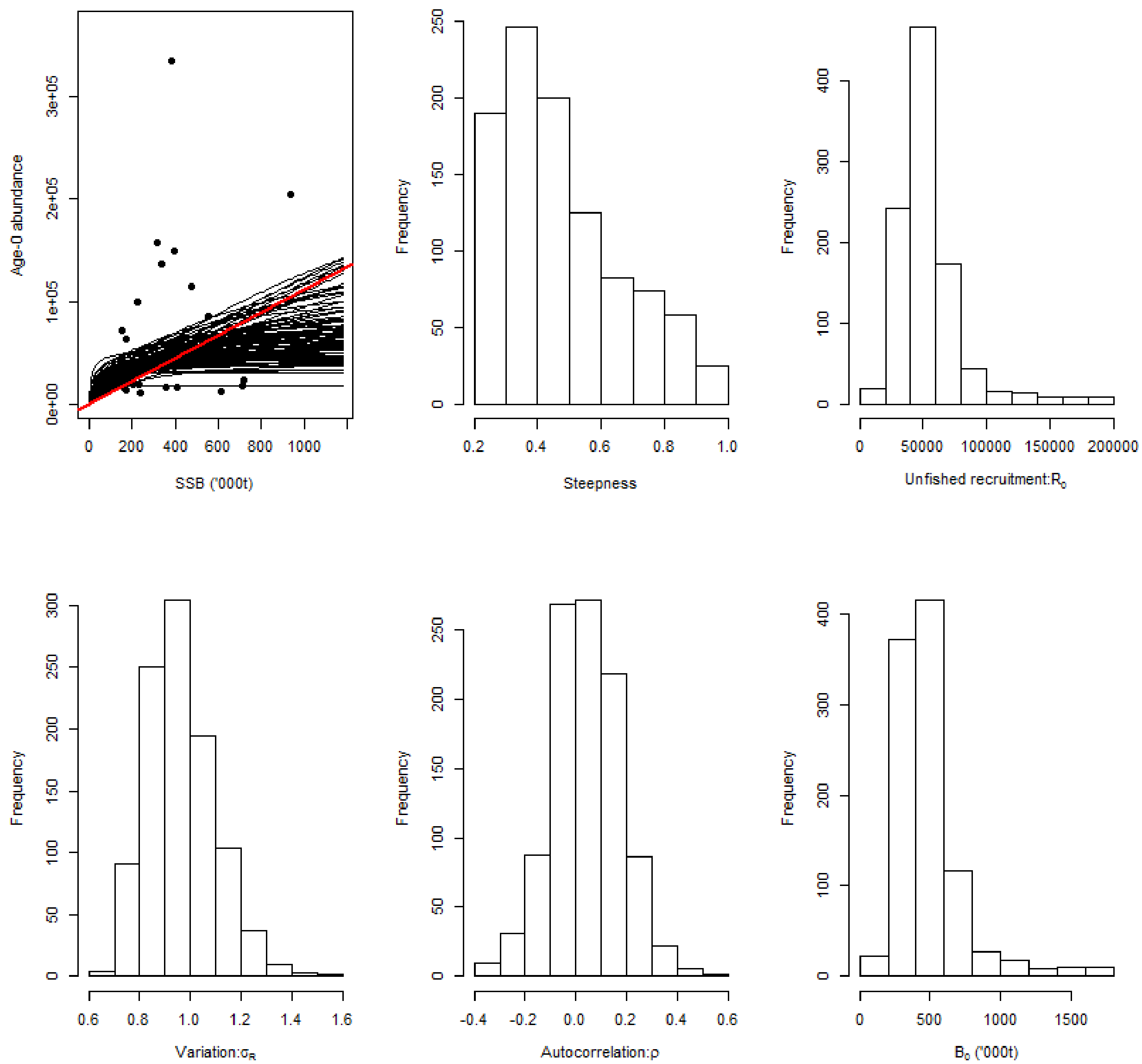

Figure 1. Diagnostic plots for the “Beverton-Holt with auto-correlation” analysis.

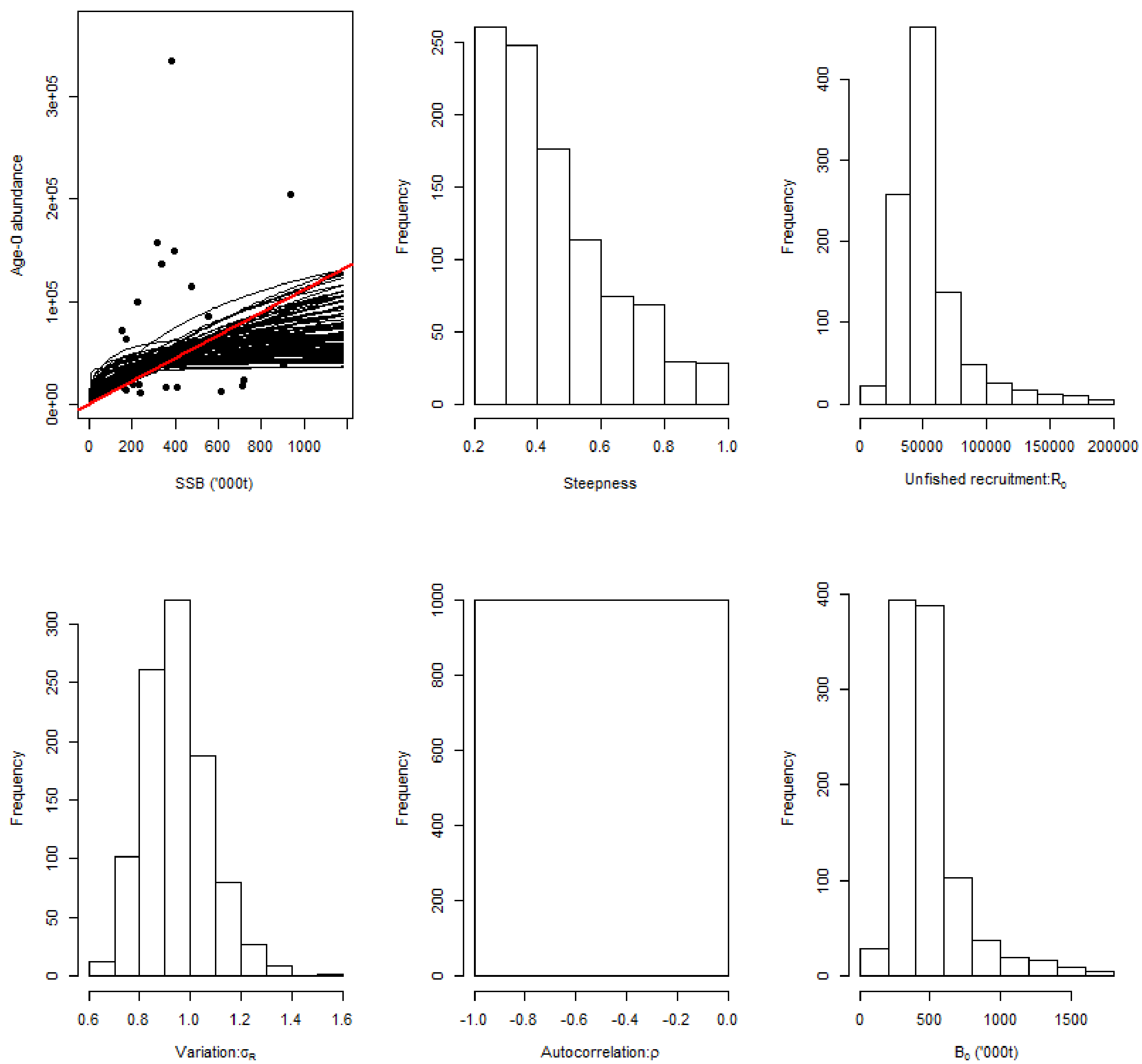

Figure 2. Diagnostic plots for the “Beverton-Holt no auto-correlation” analysis.

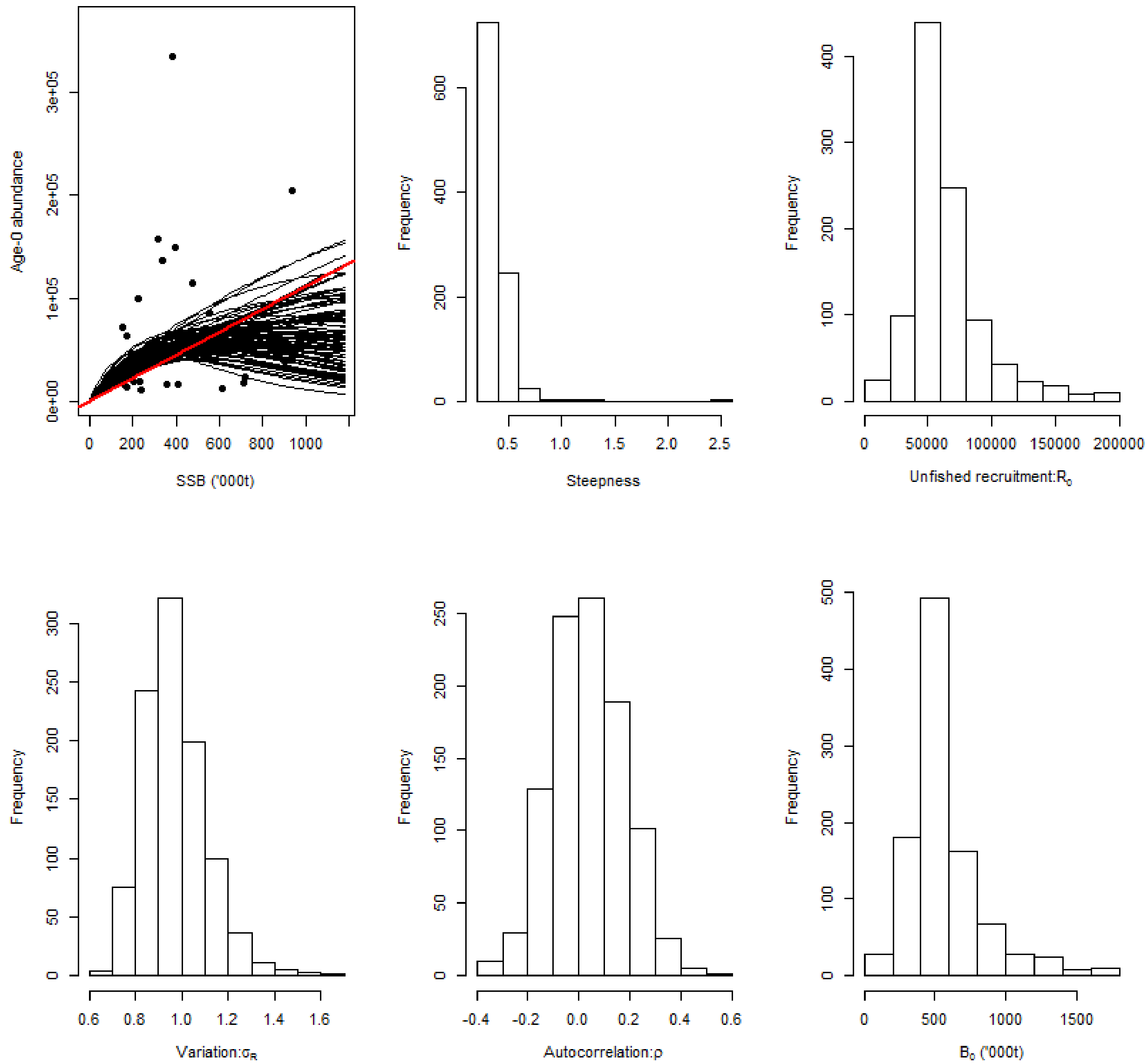

Figure 3. Diagnostic plots for the “Ricker with auto-correlation” analysis.

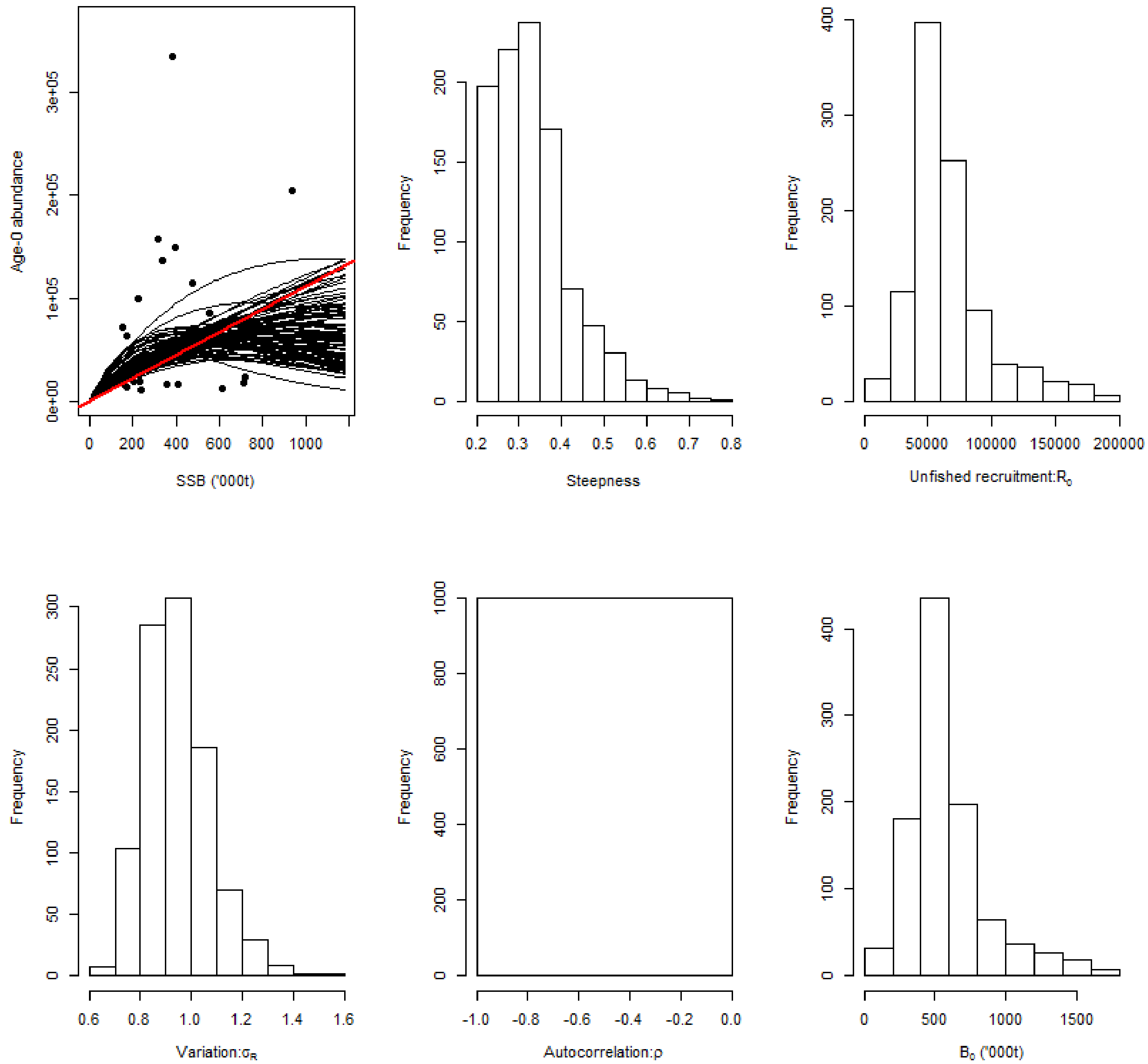

Figure 4. Diagnostic plots for the “Ricker no auto-correlation” analysis.

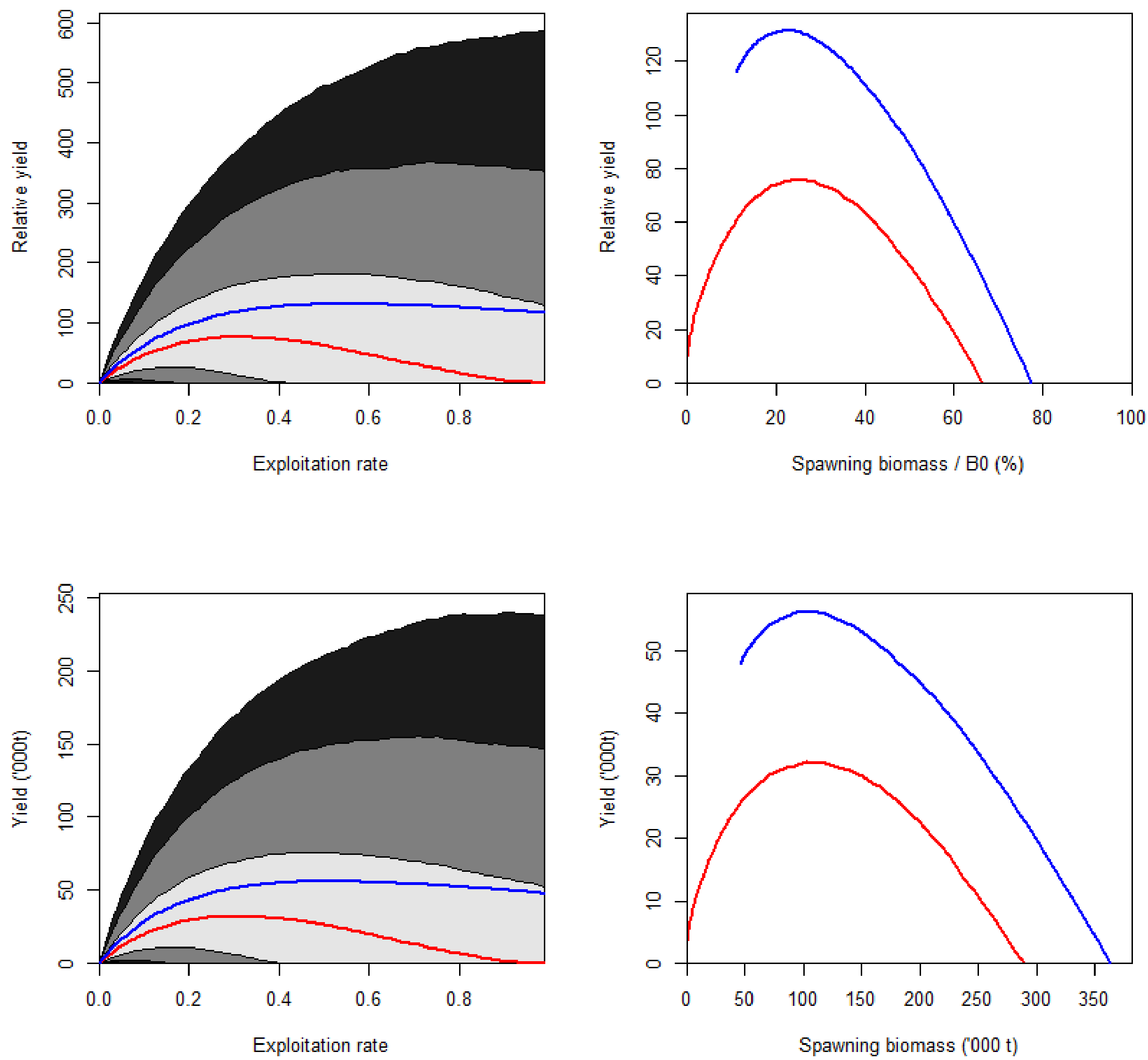

Figure 5. Yield curves (upper panels relative R_0; lower panels absolute R_0): left panels yield in year 500 versus target exploitation rate; right panels median (red) and mean (blue) yield curves. The results in the plot are based on the Beverton-Holt stock-recruitment relationship fitted with a U(-0.99,0.99) prior for ρ, and the biomass to which E_{MSY} applies is assumed to be spawning biomass.

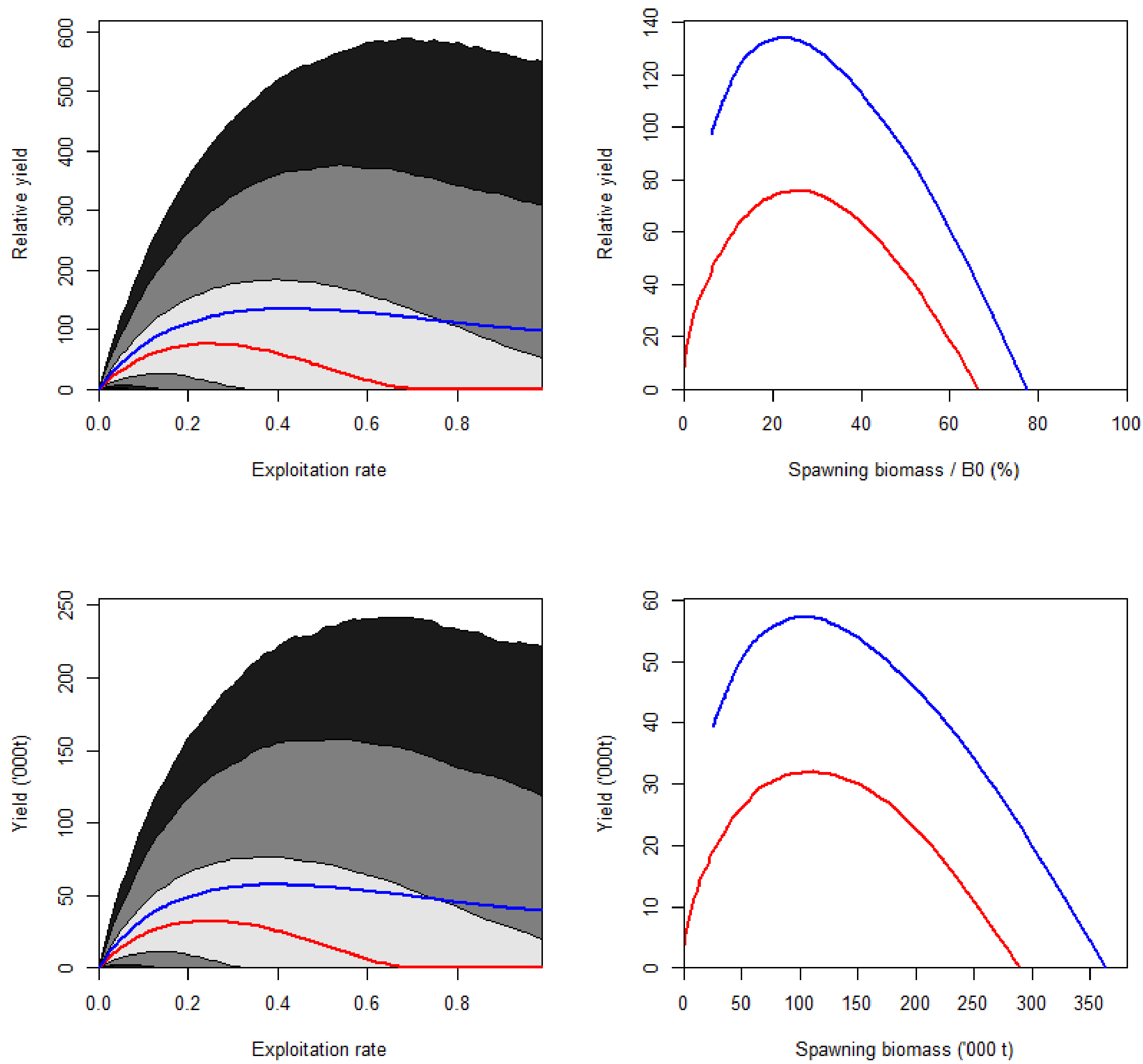

Figure 6. As for Figure 5, except that the biomass to which E_{MSY} applies is assumed to be 1+ biomass.

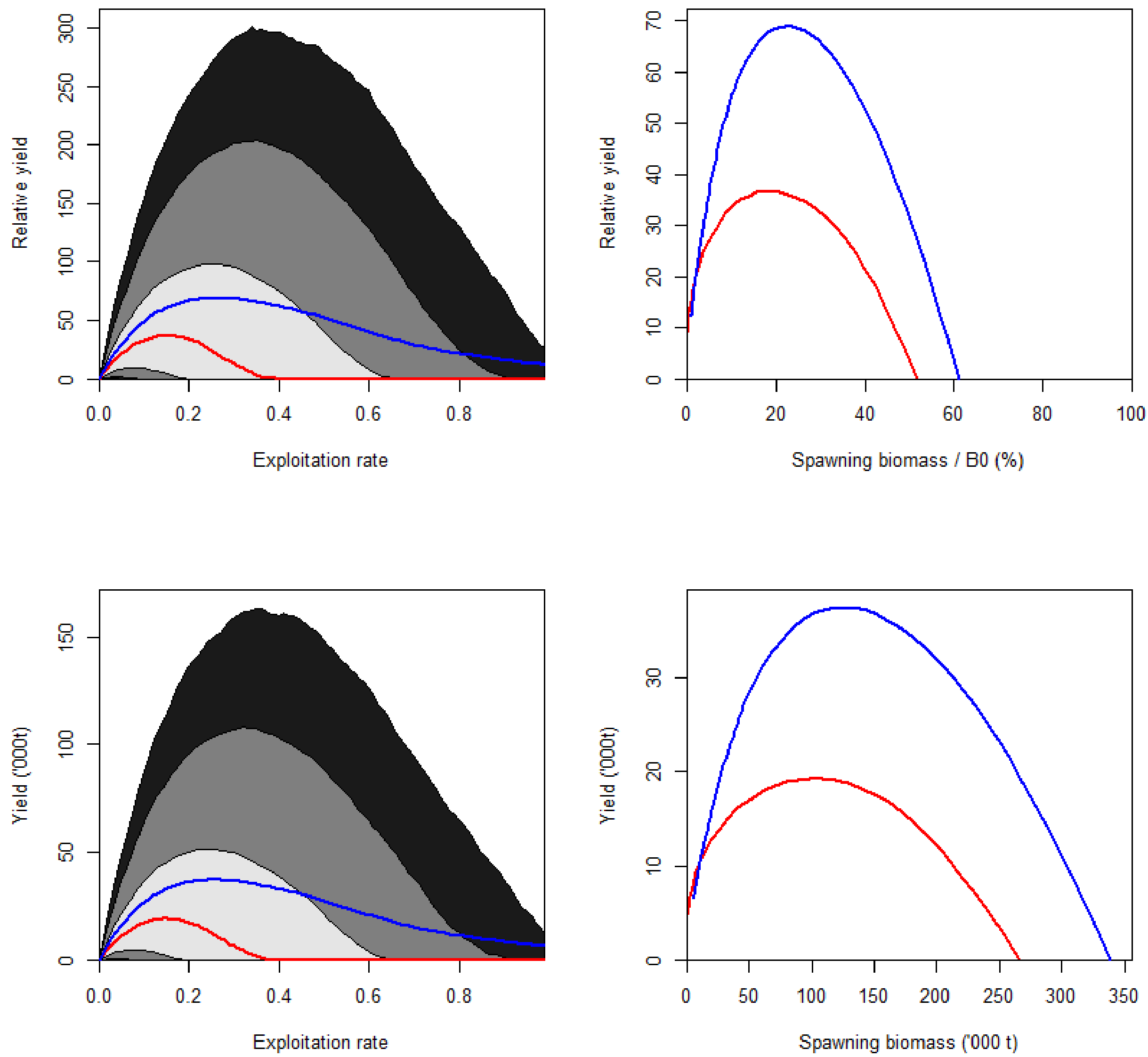

Figure 7. Yield curves (upper panels relative R_0; lower panels absolute R_0): left panels yield in year 500 versus target exploitation rate; right panels median (red) and mean (blue) yield curves. The results in the plot are based on the Beverton-Holt stock-recruitment relationship fitted with a U(-0.99,0.99) prior for ρ, and the biomass to which E_{MSY} applies is assumed to be spawning biomass.

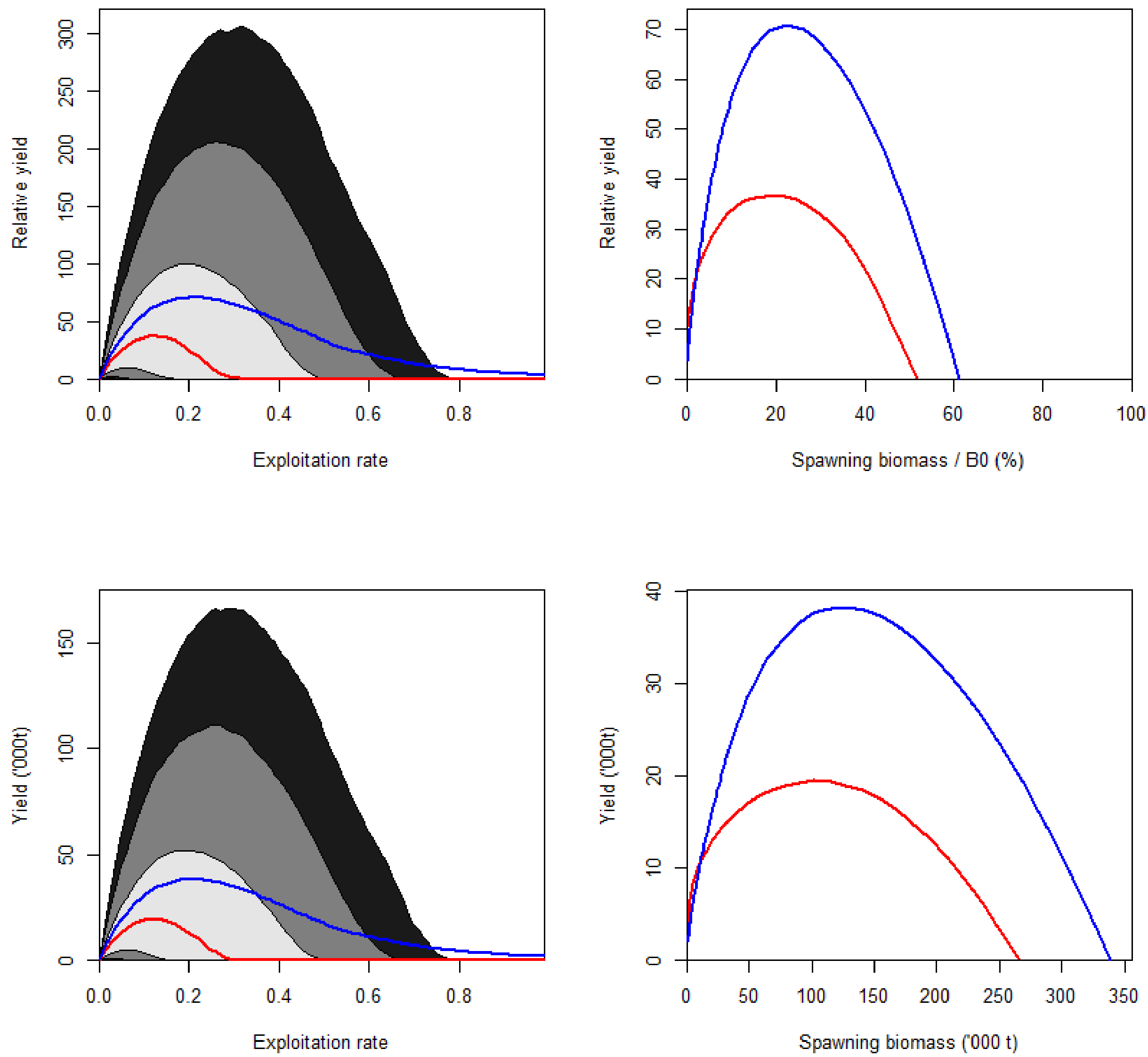

Figure 8. As for Figure 7, except that the biomass to which E_{MSY} applies is assumed to be 1+ biomass.