Assessment and Management advice for Pacific hake in U.S. and Canadian waters in 2008

Steven Martell
University of British Columbia
Fisheries Centre
2202 Main Mall
Vancouver, BC, Canada

February 19, 2008

First prepared for the 2008 joint Canada/USA Pacific hake stock assessment review meeting in Seattle WA, February 11-14, 2008. Amendments to the original document are in response to peer review comments during the review process.

Executive summary

This is an alternative assessment model (TINSS) that directly estimates the management variables C^*(the maximum sustainable yield) and F^*(the fishing mortality rate that produces C^*). The model was implemented in the AD Model Builder software and is based on the methods in Martell et al. (in press). The structural assumptions are similar to that of SS2: a Beverton-Holt stock recruitment relationship is assumed, it is assumed that the population was at an unfished state in 1966, and the model is conditioned on historical catch information. The data for TINSS was greatly simplified in comparison to SS2, where catch and catch-age information from U.S. and Canadian fisheries are aggregated into a single fishery and the selectivity curves for this aggregate fishery is asymptotic. I also assume an asymptotic selectivity curve for the fisheries independent acoustic trawl survey. In contrast to previous assessments, the assessment attempts to reduce the amount of prior information on key population parameters that ultimately define the harvest control rule and provide catch advice.

In summary the estimate of spawning stock depletion (male and female) in 2007 is 46% and recent fishing mortality rates are below F^*(Table 1). The spawning stock depletion at the start of 2008 is estimated at 43% and the 5% and 95% quantiles for the spawning stock depletion is 0.21 and 0.72, respectively. Estimates of the male and female spawning stock biomass at the start of 2008 range from 0.95 to 4.804 million mt with a median estimate of 2.235 million mt. Recent trends in fishing mortality rates have been increasing owing to the disappearance of the 1999 year class and above average landings in the commercial fisheries. Estimates of fishing mortality in 2007 range from 0.105 to 0.529 with a median value of 0.223.

Catch advice is based on a risk profile using the probability of exceeding the target fishing morality rate (F^*), probability of a decline in the 2009 spawning stock biomass and the probability of the spawning stock biomass falling below SB_{MSY}, 40% and 25% of the unfished levels (Table 2). Arbitrary levels of probability we defined for risk averse (P=0.25), risk neutral (P=0.5) and risk prone (P=0.75). Based on the risk neutral policy of not exceeding the fishing mortality, a recommended ABC for the 2008 Pacific hake fishery is 364,000 mt; the risk averse policy calls for an ABC of 264,000 mt.

In summary, catch options in excess of 300,000 mt result in a fairly significant probability of overfishing ($P \geq 0.3$), further declines in spawning stock biomass in 2009, and a significant probability of reducing the spawning stock biomass below $\mathrm{SB}_{\mathrm{MSY}}$ ($P \geq 0.4$). Catch options less than 300,000 mt result in a low probability of the spawning stock biomass falling below SB_{25} level ($P \leq 0.15$).

Table 1: Median estimate and 5% and 95% confidence intervals for the spawning stock biomass (million mt), spawning stock depletion, and fishing mortality rates in 1966 and recent years. These estimates are based on sampling the joint posterior distribution using MCMC, chain length 2,000,000 with systematic samples drawn every 200 iterations.

	Spawning stock biomass			**Depletion**			**Fishing Mortality**		
Year	median	5%	95%	median	5%	95%	median	5%	95%
1966	5.208	3.999	7.474	1.000	1.000	1.000	0.047	0.033	0.060
2003	5.027	3.342	8.509	0.968	0.725	1.310	0.146	0.078	0.253
2004	4.447	2.884	7.586	0.855	0.629	1.176	0.165	0.089	0.293
2005	3.371	2.075	5.990	0.648	0.453	0.926	0.180	0.097	0.320
2006	2.756	1.567	5.202	0.529	0.340	0.806	0.236	0.121	0.461
2007	2.432	1.210	4.892	0.468	0.263	0.746	0.223	0.105	0.529
2008	2.235	0.950	4.804	0.431	0.211	0.727			

Table 2: Decision table for catch advice. The risk level represents the probability of exceeding a specified management target for a given ABC option. The interpretation of this table is as follows; if the management goal is not to exceed the target fishing mortality rate of F^*in 2008 with a 0.25 probability, then the ABC option should be set at 0.264 million mt or less. If the management target is prevent further decline in spawning stock biomass with a 0.5 probability then the ABC should be set at 0.122 million mt or less.

Risk level	$F_{2008} \leq F^*$	$SB_{2009} \geq SB_{2008}$	$SB_{2009} \geq SB_{MSY}$	$SB_{2009} \geq SB_{40}$	$SB_{2009} \geq SB_{25}$
0.25	0.264	0.000	0.008	0.000	0.546
0.50	0.364	0.122	0.464	0.285	0.866
0.75	0.465	0.318	0.920	0.777	1.186

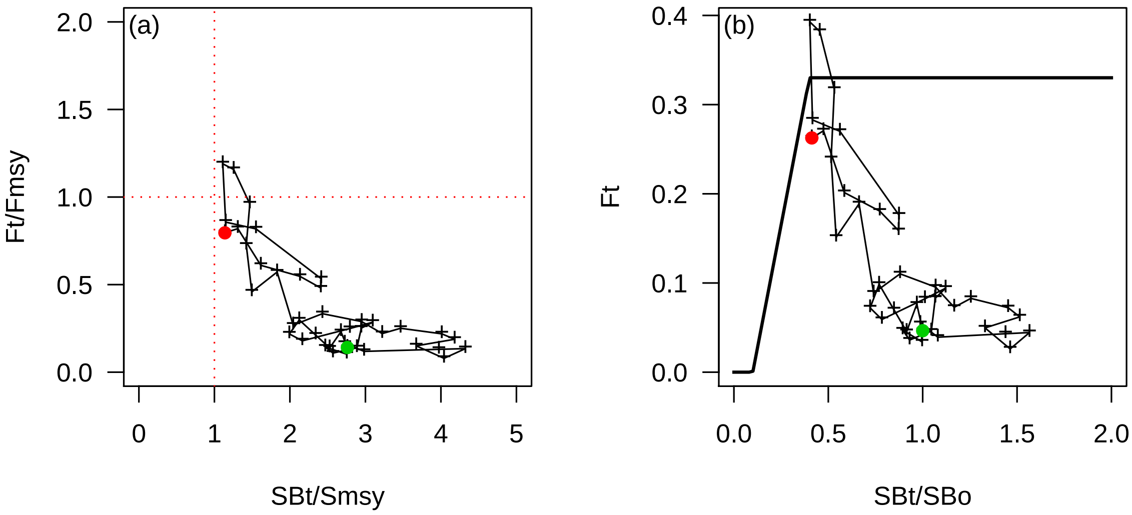

Figure 1: Maximum likelihood estimates of the spawning stock biomass relative to the unfished spawning stock biomass versus the fishing mortality rate relative to F^*(a). In panel (b) the inferred harvest control rule (thick line) and the spawning stock biomass depletion levels versus maximum likelihood estimates of historical fishing mortality rates. Green circles indicate the start of the series (1966) and red indicates the end of the series (2007).

1 Introduction

Previous assessments of Pacific hake (*Merluccius productus*) have been troubled by the lack of contrast in the acoustic survey data that allow for the estimation of the unfished biomass (B_o) and the steepness of the stock recruitment relationship. To cope with the lack of information in the acoustic survey data, the assessments have proceeded by fixing the value (h) of steepness for stock recruitment relationship and presented two alternative scenarios for the acoustic survey scaling parameter q. Fixing these parameters is necessary due to the lack of contrast in the acoustic survey data; however, it also results in a gross underestimation of the uncertainty in model results and estimates of the reference points used in the determination of Acceptable Biological Catch (ABC).

At present, uncertainty in parameters that define the harvest control rule is only represented by the uncertainty associated with size selectivity parameters in the various commercial fisheries as well as the acoustic survey itself. The parameters that define the underlying production function include the instantaneous natural mortality rate (M), the steepness of the stock recruitment relationship (h) and a measure of population scale (usually the unfished spawning stock size or B_o). In previous assessments, h and M are fixed, and the population scale is determined by the combined effects of selectivity in the acoustic survey and the survey scaler q (which is fixed at two different values). For example for a given value of q, estimates of the unfished biomass increase as the acoustic survey selectivity becomes more dome-shaped, and vice-versa.

Historically, management advice is based on the application of the 40-10 harvest control rule. Three critical pieces of information are required to apply the harvest control rule: 1) an estimate of F_{MSY} and B_{MSY} which is approximated by F_{40} and B_{40}, respectively, 2) an estimate of the current level of depletion in the spawning stock biomass, and 3) a biomass forecast based on historical recruitment or the underlying stock recruitment relationship. Accurate estimates of F_{MSY} require accurate estimates of M, h, which are difficult to obtain in many (if not all) fisheries assessments; therefore the a proxy F_{40} (which is the fishing mortality rate that reduces the spawning potential ratio to 40% of its unfished state) is often used to approximate F_{MSY}. This approximation has been shown to achieve nearly 80% of the maximum yield over a wide range of stock recruitment parameters with a variety of stock recruitment models (Clark, 1991, 2002). Similarly, B_o is also difficult to estimate in many cases; therefore the spawner potential ratio (SPR) is used as a measure of depletion. The current level of depletion is determined by comparing the ratio of present day spawning biomass to the estimated unfished spawning biomass. Finally, the forecast is based current levels of depletion and estimates of h.

There are a few unresolved problems and inconsistencies in the input data for SS2 or any other age-structured model. First there is a large inconsistency between information in the age-compositions and the acoustic survey biomass index. The age compositions suggest a buildup of biomass through the late 1980s owing to the strong 1980 and 1984 cohorts, yet the biomass index is relatively flat during this time period.

In contrast to previous assessments for Pacific hake, this assessment attempts to reduce the amount of prior information that is used on key population parameters that ultimately defines the harvest control rule and catch advice. To do this, I have implemented a age-structured model that is parameterized from a management oriented perspective, where the leading parameters are $C^*$and F^*. The population model is structurally similar to that of SS2, where I assume that the stock is at its unfished state in 1966, recruitment follows a Beverton-Holt stock-recruitment relationship, and the model is conditioned on the historical catch information. The fundamental differences between the two approaches is that I make no prior assumptions about the survey q, and no direct prior assumptions about the steepness of the stock recruitment relationship. The model parameterization is such that there is an implied prior for the steepness of the stock recruitment function; however, this prior is very diffuse in comparison to 2008 SS2 implementation. Another fundamental difference is the treatment of the data. In this application, catch data from U.S. and Canadian operations are aggregated into a single fishery, and it is assumed that selectivity curve for the aggregate fishery and the acoustic trawl survey is asymptotic.

2 Methods

A summary of the input data and complete technical description of the model is provided in Appendix A and B, respectively. For technical details on the acoustic trawl surveys, please refer to Fleischer et al. (2005). For a more detailed description of the fishery and historical management of the fishery see Helser and Martell (2007) for more details. The purpose of this section is three fold: 1) summarize the modeling approach, 2) provide documentation for informative prior distributions, and 3) provide a technical description on how the reference points and catch advice is formulated.

2.1 Modeling approach

The principle difference between the assessment here, and that of last years assessment using Stock Synthesis II (SS2), is that the leading parameters in this model pertain to the management parameters F^*(the fishing mortality rate that produced the maximum sustainable yield) and C^*(the maximum sustainable yield). Whereas, SS2 estimates the unfished biomass B_o and the steepness of the stock recruitment relationship h; these parameters are then transformed into the management variables F_{40} and MSY.

The approach was to fit and age-structured population dynamics model to time series information on relative abundance, proportions-at-age in the commercial fishery, and proportions-at-age from the acoustic trawl survey index using a Bayesian estimation framework. The commercial catch and age-composition information from Canada and the U.S. has been combined to represent a single fishery. The aggregation of the commercial catch data has the potential to create a bias in the predicted-age composition because it assumes that the age-specific fishing mortality rates between the two countries has been relatively consistent over time.

The objective function contains 5 major components: 1) the negative loglikelihood of the relative abundance data, 2) the negative loglikelihood of the catch-at-age proportions in the commercial fishery, 3) the negative loglikelihood of the catch-at-age proportions in the acoustic survey, 4) the prior distributions for model parameters, and 5) two penalty functions that constrain the estimates of steepness to lie between 0 and 1, and prevent exploitation rates exceeding 1. Note that the value of the penalty functions was 0 for all samples from the posterior distribution. The joint posterior distribution is defined by equation (T17.14). This distribution was numerically approximated using the Markov Chain Monte Carlo routines built into AD Model Builder (Otter Research, 1994). Posterior samples were drawn systematically every 400 iterations from a chain of length 2,000,0000 (the first 1000 samples were dropped to allow for sufficient burnin). Convergence was diagnosed using various test provided in the R-package CODA (R Development Core Team, 2006), as well as, running medians and visual inspection of the trace plots. Where possible, we provide comparisons between the maximum likelihood estimates and median estimates from the marginal posterior distributions. Catch advice is based on the samples from the joint posterior distribution (T17.14).

2.1.1 Assumptions

There is no prior assumption about the scaling parameter for the acoustic biomass survey (q), and the index was treated as a relative abundance index that is directly proportional to the vulnerable biomass as seen by the acoustic survey. It is assumed that the observation errors in the relative abundance index are lognormally distributed. Fishing mortality in the assessment model is conditioned on the observed total catch weight (combined US and Canada catch), and it is assumed that total catch is known and reported without error. Age-composition information is assumed to come from a multinomial distribution where the predicted proportion-at-age is a function of the predicted population age-structure and the age specific vulnerability to the fishing gear. The effective sample size of the age-composition is used a measure of the observation or sampling error variance. Effective sample sizes were determined through a joint process of iterative re-weighting, retrospective analysis, and comparison of the estimated variances and mean squared errors. No aging errors were assumed in this assessment.

Historical observations on mean weight-at-age shows systematic changes, where the average weights-at-age have declined from the mid 1970s and increased again slightly late 1990s (Figure 2). A number of the historical cohorts have a growth trajectories that initially increase from age-2 to age-8 then decline or stay relatively flat (e.g., 1977 cohort in Figure 2). Given these data, there are at least three alternative explanations for the observed decreases in mean weight-at-age: 1) changes in condition factor associated with food availability, 2) intensive size selective fishing mortality with differential fishing mortality rates on faster growing indivuals, and 3) apparent changes in selectiviity over time (e.g., dome-shaped selectivity) where there is a low probability of capturing faster growing fish. All three of these variables are confounded, and it is not possible to capture decreasing weight-at-age using the von Bertalanffy growth model and a fixed allometric relationship between length and weight. As such, the assessment model herein uses the observed mean weight-at-age data from the commercial fishery to scale population numbers to biomass.

The structural assumptions of the model assume that recruitment follows a Beverton-Holt type model and the process error terms are represented by a vector of deviation parameters that are assumed to be lognormally distributed. Both fishing mortality and natural mortality are assumed to occur simultaneously; instantaneous fishing mortality is based on the Baranov catch where the analytical solution for F_t is found using an iterative method. Selectivity, or vulnerability-at-age, to the fishing gear is assumed to be age-specific, time-invariant, and is represented by an asymptotic function (T15.5).

2.2 Prior distributions

The underlying production function is defined by three key population parameters (C^*, F^*, and M) and the parameters that define age-specific selectivity ($v_a = f(\hat{a}_h, \hat{\gamma})$). Informative lognormal prior distributions were used for C^*, F^*, and M where the log means and log standard deviations are given in Table 3. These prior distributions were developed on an *ad hoc* basis and not necessarily derived from meta-analytic work that is the typical source of prior information.

The global scaling parameter in this model is C^*; the maximum long-term sustainable yield. Since 1966, the average annual landings removed from this population is 218,963.5 mt, and in the last decade 282,408.7 mt. We assume a rather diffuse prior for $C^*$with mean corresponding to 200,000 mt and a standard deviation of 396,000 mt. This represents a 95% confidence interval of roughly 138,000 mt to 652,000 mt. Assigning a prior density for $C^*$is nearly equivalent to assigning a prior density for the global scaling parameter q.

Table 3: Prior distributions for model parameters.

Parameter	prior density	range	μ	σ	a	b
C^*	lognormal	(0.01-3.0)	0.2	0.396		
F^*	lognormal	(0.01-0.9)	0.35	0.262		
M	lognormal	(0.05-0.9)	0.23	0.1		
$\hat{a}, \bar{a}$	uniform	(0.0-14.0)				
$\hat{\gamma}, \bar{\gamma}$	uniform	(0.05-5.0)				
ρ	beta	(0.01-0.99)			3.5	31.5
φ	inverse gamma	(0.02-100)			7.5	5.78

A lognormal prior was assumed for M with a mean corresponding to 0.23 (which is the assumed fixed value in Helser and Martell (2007)) and a standard deviation of 0.1. This roughly corresponds to a 95% confidence interval of 0.19 and 0.28 for M, which is lower than the range reported in (Bailey et al., 1982, Table 10).

Uniform prior distributions were assumed for the selectivity parameters for the commercial fishery and the acoustic trawl survey. These parameters are bounded between 0 and 14

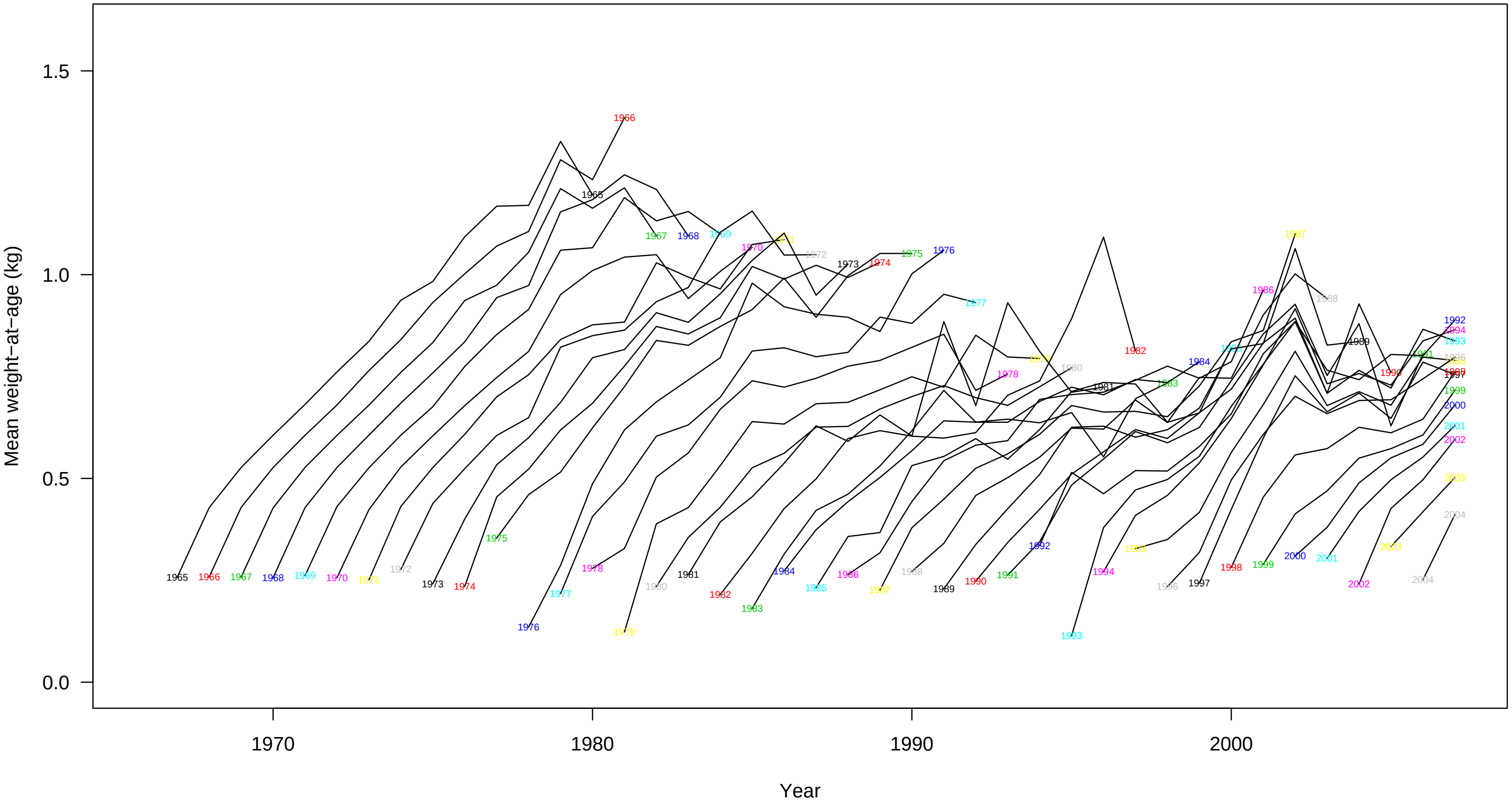

Figure 2: Observed mean weights-at-age by cohort in the commercial catch. Text labels for each line represent the cohort year.

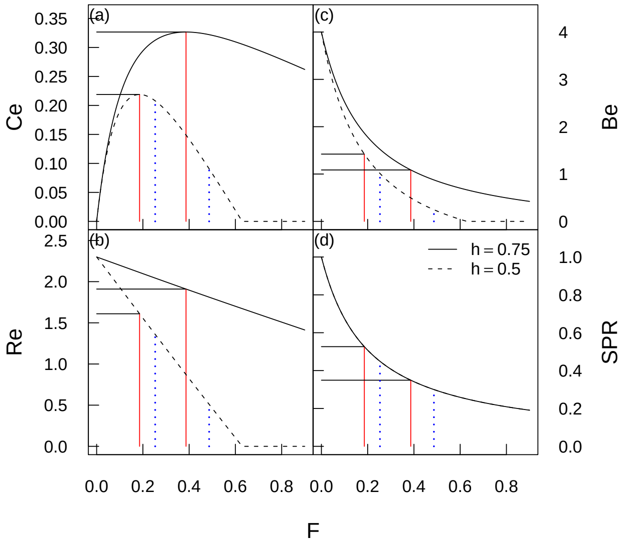

Figure 3: Relationship between equilibrium fishing mortality rate and yield (a), recruitment (b), biomass(c) and spawner per recruit(d) with an assumed value of $h = 0.75$ and $h = 0.5$. The vertical lines in each panel represent estimates of F^*(solid lines), F_{45}, and F_{30} (dotted lines). Note that the y axis scaling is arbitrary (i.e. B_o was assumed at 4 units of biomass).

years for the age at 50% vulnerability and 0.05 and 5.0 for the standard deviation in age at 50% vulnerability.

In comparison with Helser and Martell (2007), a prior probability for $F^*$is nearly equivalent to a prior probability for steepness h. A lognormal prior was assumed for F^*, with a mean corresponding to 0.35 and a standard deviation of 0.262 (corresponds to a 95% confidence interval of 0.21 and 0.59). To derive the prior for F^*, a steady state age-structured model was developed to calculate spawning potential ratio based on growth parameters from Francis et al. (1982), a natural mortality rate of 0.23, and a logistic selectivity curve ($\hat{a} = 3.13, \hat{\gamma} = 0.8$). Arbitrarily, it was assumed that production is maximized somewhere between SPR=0.3 and SPR=0.45, and the corresponding values for F_{30} and F_{45} were then calculated. Based on the growth-maturity, natural mortality, and assumed selectivity the values correspond to $F_{30} = 0.48$ and $F_{45} = 0.25$, which were then assumed to be the 10th and 90th percentiles for a lognormal distribution. Note that the Spawning potential ratio curve is insensitive to the assumed value of steepness (Figure 3) and that F_{40} is the assumed proxy for F^*that is used by the Pacific Fisheries Management Council.

The transition from $(C^*,F^*) \Rightarrow (B_o, h)$, that is carried out using the algorithm described in Table 15, implies a prior density for the steepness parameter in the stock recruitment relationship. The implied prior density for h used in this assessment is shown in Figure 4. Note that in the Beverton-Holt stock recruitment model, values of h range between 0.2 and 1.0, where 0.2 implies that recruitment is nearly proportional to spawner/egg production, and 1.0 implies that recruitment is the same when spawner/egg production is reduced to 20% of its unfished state. The implied prior for h is sensitive to two key model components: the assumed prior distribution for F^*, and the age at which fish recruit to the fishery relative to the age at which fish mature. Larger values of F^*imply a more productive stock and higher values of h for given selectivity and maturity schedules. Similarly, if fish recruit to the fishery prior to maturing then the levels of recruitment compensation (or h) must increase for a given value of F^*. Therefore, a critical piece of information is the maturity-at-age and weight-at-age schedules used to develop the age-specific fecundity relationship.

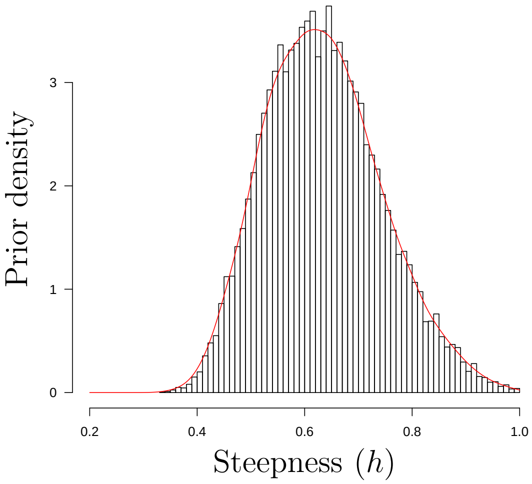

Figure 4: Implied prior for the steepness parameter in the stock recruitment relationship. Note that steepness is derived from the leading parameters Θ; therefore, any assumed prior information for Θ results in an implied prior for derived quantities such as h.

2.3 Reference points and catch advice

Catch advice in this model is based on a modified 40:10 harvest control rule, where the modification is to fish at F^*, rather than F_{40}. Unless otherwise stated, the reference point calculations and catch advice is based on the most recent information about growth (Table 13) and maturity-at-age information from Dorn and Saunders (1997).

The reference points for the harvest control rule are $F^*$and SB_{40}. Recall that $F^*$is the fishing mortality rate that produces the maximum sustainable yield, and this differs from that assumed in the previous assessments where F_{40} was used. $F^*$is estimated as a leading parameter, and $\text{SB}_{40}$is 40% of the unfished spawning biomass (SB_o). An alternative (but as it turns out, less conservative) harvest rule would be to use SB_MSYas the reference point in the harvest control rule, where SB_MSY=$R_e\phi_e$ evaluated at $F^*$and C^*.

Catch advice was generated by projecting the stock abundance forward to 2009 by applying catch options between 0 and 750,000 mt tons over 25 equally spaced intervals and then calculating various management objectives for each of the 5,000 samples from the joint posterior distribution. It was assumed in each simulated projection that the total catch option was fully utilized and implemented without error. In the stock projections, age-1 recruits for 2006-2009 were generated using the underlying Beverton-Holt stock recruitment model with annual lognormal recruitment deviates with standard deviation equal to the current estimate of standard deviation in the process errors (τ).

A decision table for catch advice (ABC options) was developed using measures of overfishing (probability that the ABC option will result in a fishing mortality rate that exceeds F^*), and four measures of spawning stock depletion. The first measure is the probability that the spawning stock biomass in 2009 will be greater than the spawning stock biomass in 2008, and the second measure is the probability that the spawning stock biomass will be greater than $\mathrm{SB}_{\mathrm{MSY}}$. The third measure is the probability that the spawning stock biomass will be greater than SB_{40}, and the fourth measure is the probability that the spawning stock biomass will remain above SB_{25}. For each sample from the joint posterior distribution the projection model loops over 25 increments of this ABC ranging from 0 to 750,000 mt and then calculates the corresponding fishing mortality rates and levels of spawning stock depletion. We then score the fishing rate and spawning stock depletion on a 0 or 1 scale (0 not overfishing or spawning stock biomass greater than or equal to management target) and fit a binomial (link logit) model versus ABC option to these data. The result is a sigmoid like curve or the cumulative probability of an ABC option versus management objective can be assessed. For specified levels of risk, ABC options for each management objective are then provided in a decision table. This cumulative probability distribution is also compared to the cumulative density function of catch advice produced by the 40/10 harvest control rule.

3 Results

Maximum likelihood estimates of the vulnerable biomass, fishing mortality rates, age-1 recruits and historical landings are summarized in Fig. 5. During the late 1960 and 1970s, annual landings averaged 169,000 tons and the corresponding fishing mortalities were less than 0.08 per year. During the 1980s catches increased from 90,000 tons to just over 300,000 tons and the fishing mortality rates during this period averaged less than 0.06 per year. Two exceptionally strong cohorts (1980, 1984) were responsible for a large increase in the vulnerable biomass during this time period. The vulnerable biomass peaked in the mid 1980s declined steadily to a low of 1.46 million tons in 2000. During this time period, there were no significant recruitment events (Fig. 5c), and also during this time period annual landings increased from 110,000 tons in 1985 to nearly 312,000 tons in 1999. The 1999 cohort was an exceptional year class, and the vulnerable biomass nearly doubled from 1.42 millon tons in 2000 to 2.94 million tons in 2004.

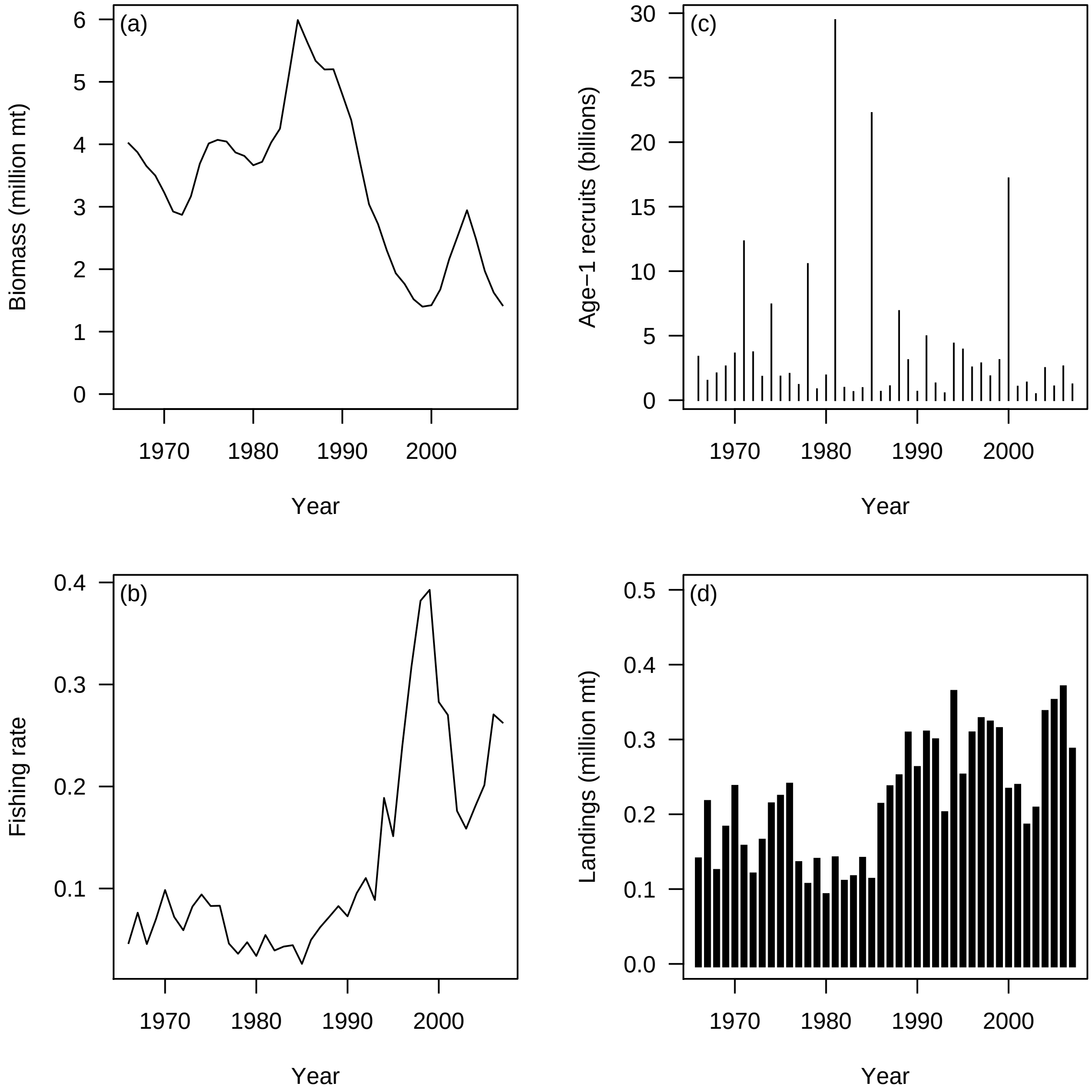

Figure 5: Maximum likelihood estimates of vulnerable biomass (panel a), fishing mortality (b), age-1 recruits (c) and the observed historical landings (d) for U.S. and Canadian fisheries combined.

Table 4: Maximum likelihood estimates of vulnerable biomass (B_t), make and female spawning biomass (SB_t), , landings (C_t millions mt), instantaneous fishing mortality rates (F_t), 2+ and 3+ biomass ($B_{t,2+}$, $B_{t,3+}$), and total catch over 2+ and 3+ biomass ($C_t/B_{t,2+}$, $C_t/B_{t,3+}$), from 1966 to the begining of 2008.

Year	B_t	SB_t	SB_t/SB_0	C_t	F_t	$B_{t,2+}$	$B_{t,3+}$	$C_t/B_{t,2+}$	$C_t/B_{t,3+}$
1966	4.02	5.08	1.00	0.14	0.05	5.96	5.30	0.02	0.03
1967	3.87	4.92	0.97	0.21	0.08	5.80	5.15	0.04	0.04
1968	3.65	4.65	0.91	0.12	0.05	5.23	4.94	0.02	0.02
1969	3.50	4.31	0.85	0.18	0.07	4.82	4.41	0.04	0.04
1970	3.23	3.91	0.77	0.23	0.10	4.52	4.01	0.05	0.06
1971	2.92	3.66	0.72	0.15	0.07	4.47	3.76	0.03	0.04
1972	2.87	3.98	0.78	0.12	0.06	6.27	3.89	0.02	0.03
1973	3.17	5.14	1.01	0.16	0.08	6.83	6.13	0.02	0.03
1974	3.69	5.70	1.12	0.21	0.09	6.50	6.12	0.03	0.03
1975	4.01	5.41	1.07	0.22	0.08	6.75	5.40	0.03	0.04
1976	4.07	5.42	1.07	0.24	0.08	6.32	5.99	0.04	0.04
1977	4.04	5.31	1.05	0.13	0.05	6.04	5.49	0.02	0.02
1978	3.87	4.73	0.93	0.10	0.04	5.03	4.91	0.02	0.02
1979	3.81	4.54	0.89	0.14	0.05	6.05	4.32	0.02	0.03
1980	3.66	5.06	1.00	0.09	0.03	6.10	5.92	0.01	0.02
1981	3.72	5.02	0.99	0.14	0.05	5.29	5.11	0.03	0.03
1982	4.03	5.49	1.08	0.11	0.04	9.91	4.70	0.01	0.02
1983	4.25	7.31	1.44	0.11	0.04	9.63	9.44	0.01	0.01
1984	5.11	7.95	1.57	0.14	0.04	8.48	8.37	0.02	0.02
1985	5.99	7.43	1.46	0.11	0.03	7.57	7.44	0.01	0.01
1986	5.66	6.76	1.33	0.21	0.05	10.56	5.98	0.02	0.04
1987	5.34	7.69	1.51	0.23	0.06	9.49	9.38	0.02	0.02
1988	5.20	7.38	1.45	0.25	0.07	7.87	7.65	0.03	0.03
1989	5.20	6.37	1.26	0.31	0.08	7.43	6.26	0.04	0.05
1990	4.80	5.93	1.17	0.26	0.07	6.99	6.36	0.04	0.04
1991	4.39	5.43	1.07	0.31	0.10	5.81	5.70	0.05	0.05
1992	3.70	4.47	0.88	0.30	0.11	5.32	4.39	0.06	0.07
1993	3.04	3.76	0.74	0.20	0.09	4.33	4.07	0.05	0.05
1994	2.73	3.37	0.66	0.36	0.19	3.62	3.48	0.10	0.10
1995	2.30	2.75	0.54	0.25	0.15	3.12	2.75	0.08	0.09
1996	1.94	2.62	0.52	0.31	0.24	3.61	2.81	0.08	0.11
1997	1.76	2.70	0.53	0.33	0.32	3.58	2.96	0.09	0.11
1998	1.52	2.31	0.45	0.32	0.38	2.94	2.44	0.11	0.13
1999	1.40	2.04	0.40	0.31	0.39	2.52	2.19	0.12	0.14
2000	1.42	2.11	0.42	0.23	0.28	2.80	2.14	0.08	0.11
2001	1.67	2.85	0.56	0.24	0.27	6.20	2.47	0.04	0.10

Table 4: *(continued)*

Year	B_t	SB_t	SB_t/SB_0	C_t	F_t	$B_{t,2+}$	$B_{t,3+}$	$C_t/B_{t,2+}$	$C_t/B_{t,3+}$
2002	2.16	4.44	0.87	0.18	0.18	6.11	5.87	0.03	0.03
2003	2.55	4.43	0.87	0.21	0.16	4.99	4.68	0.04	0.04
2004	2.94	3.92	0.77	0.33	0.18	4.14	4.06	0.08	0.08
2005	2.49	2.97	0.58	0.35	0.20	3.54	2.91	0.10	0.12
2006	1.97	2.41	0.47	0.37	0.27	2.79	2.58	0.13	0.14
2007	1.63	2.09	0.41	0.28	0.26	2.68	2.11	0.11	0.13
2008	1.42	1.90	0.37			2.28	2.07		

The maximum likelihood estimate of the 2008 spawning stock biomass is 1.90 millon tons, which corresponds to a depletion level of 0.37 (Fig. 6ab, Table 4). This is below the management target of 0.4. In comparison to Helser and Martell (2007), the estimated level of depletion in last years assessment was 0.309. Estimates of female spawning stock biomass in 2007 were nearly identical between this study and last years assessment (female spawning stock biomass is 1.045 and 1.103 million mt, respectively). The difference in the levels of depletion over last years assessment owes to differences in the estimates of the unfished spawning stock biomass. In this study, the maximum likelihood estimate of the unfished female spawning stock biomass is 2.54 million mt, and in last years assessment it was estimated at 3.567 million mt.

A major factor that influences estimates of the 1966 states (assumed to be the unfished state) is the relative weighting of the age-composition data and the assumed variances in the recruitment deviations and observation errors. In this assessment the total variance φ^2 is estimated and partitioned (via the estimated ρ parameter) into observation and process error components represented by the errors in the relative abundance index and recruitment deviates, respectively. Due to the age-composition information, φ^2 and ρ are estimable quantities (Deriso et al., 2007); however the assumed variance (or sample sizes) in the age composition information does influence φ^2 and ρ. In short, increasing the assumed weights on the age-composition information tends to increase the biomass peak in the mid 1980s, raises the overall population scaling, and results in greater depletion estimate.

In this assessment we assume a constant age-selectivity curve for both the commercial and acoustic surveys (Fig. 7c). This is markedly different from previous assessments where selectivity is allowed to vary over specified time blocks. In this case, we have dramatically reduced the effective sample sizes in the likelihood (T18.3) on the age-composition information to 35 for the commercial fisheries and 28 for the acoustic surveys. In short, we are willing to accept a lack of fit in the age-composition data in order to place more weight on the trend information in the biomass indices generated from the acoustic trawl surveys. This also allows for visual inspection of patterns in the residuals to determine if alternative selectivity functions (i.e., dome-shaped) would be more appropriate to explain the data, and aid in the justification of time-blocks associated with changes in selectivity (i.e., SS2 implementation).

Reasonable fits were obtained to the age-compositions in the acoustic trawl surveys (Figs.

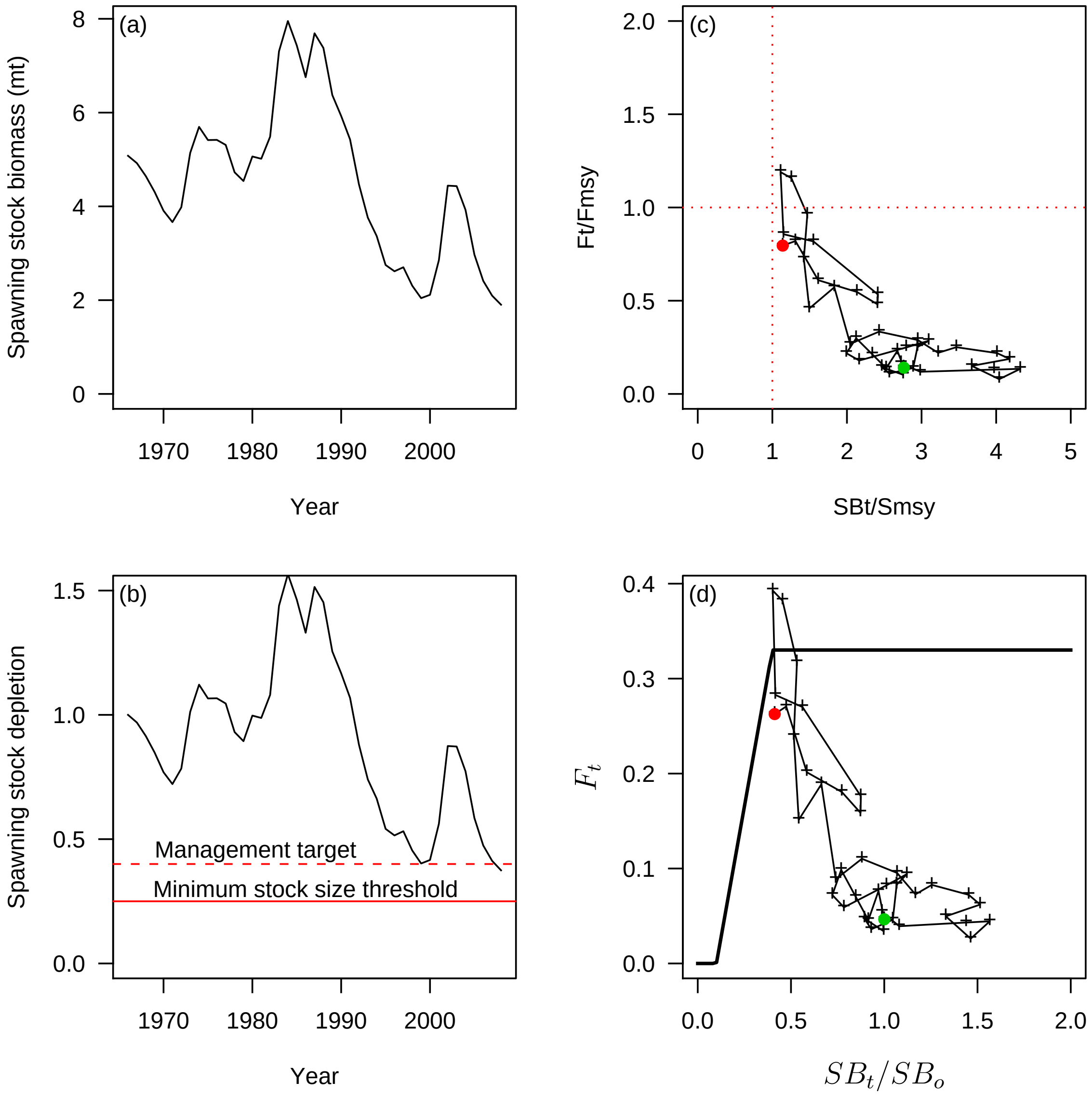

Figure 6: Maximum likelihood estimates of spawning stock biomass (a), spawning biomass depletion (b), the ratio of fishing mortality rates to C^*versus the spawning stock biomass to S_{msy} (c) and the harvest control rule (d). Note that the spawning stock biomass calculations include both male and females.

8-9). The largest residual occurred for the age-2 fish in 2007 and age-7 fish in 1977 (Fig. 9). The model also under-estimates the age-2 fish in 1995 and in 2001, corresponding to the 1993 and 1999 cohorts. The 1999 cohort was above the average long-term recruitment and the 1993 cohort is near the average long-term recruitment. In many years (e.g., 1977, 1980, 1986, and 2007, Fig. 9) there is a pattern in the pearson residuals where the model over-estimate the proportions at ages 2-5 and ages 12-14, and under-estimate the proportions at ages 6-11; this pattern would be better explained with a dome-shaped selectivity curve. But this pattern is not apparent in all years. Also, estimates of the instantaneous natural mortality rate M are substantially higher than the previously assumed values of 0.23. This increase in M better explains the disappearance of older fish in the age-composition information, given that the assumed shaped of the selectivity curves is asymptotic.

In the commercial fishery, we assumed an constant asymptotic selectivity curve and obtained surprisingly good fits to the older age-classes in the commercial catch-age proportions (Figs. 10-11), with the exception of the year 2000 (Fig. 12), and the persistent under-estimate of the proportions-at-age in the plus group. The largest residual variation in the commercial age-composition data occurred in ages 2 and 3. The model tends to under estimate the 1980 and 1984 cohorts at age-2 which would be consistent with a shift to higher selectivity for age-2 fish in 1981 and 1985. The opposite pattern was also observed for the 1999 cohort where the model tended to over-estimate the proportion at age-2. In 1988, there is a positive residual for the 1980 cohort (age-8) that tracks trough to age-12 in 1992. From 1977 to 1979 there is a negative pattern where the residuals from age-12-15; this could be explained by dome-shaped selectivity and or the initialization of the model with a stable age-distribution (with a plus group). In the rest of the time series, there are few exceptions where there are negative residual patterns for ages 10-14 (indicative of a dome-shaped selectivity curve); however, the residuals for the plus group are all negative with exception of the year 2000-2001 when the fish did not show up in the Canadian zone and the fleet operated in the northern portion of the Canadian zone.

Overall, the constant selectivity assumption fits the commercial catch-age data reasonable well (Fig 10). There is a marked pattern in the Pearson residuals that appear to correspond to an aging error pattern (Fig 12) around the strong cohorts (e.g., 1980, 1984 cohorts). Up to age-8, the model tends to over estimate the 1981 cohort and underestimate the 1979 cohort. There are persistent underestimates of the 1985 cohort as well.

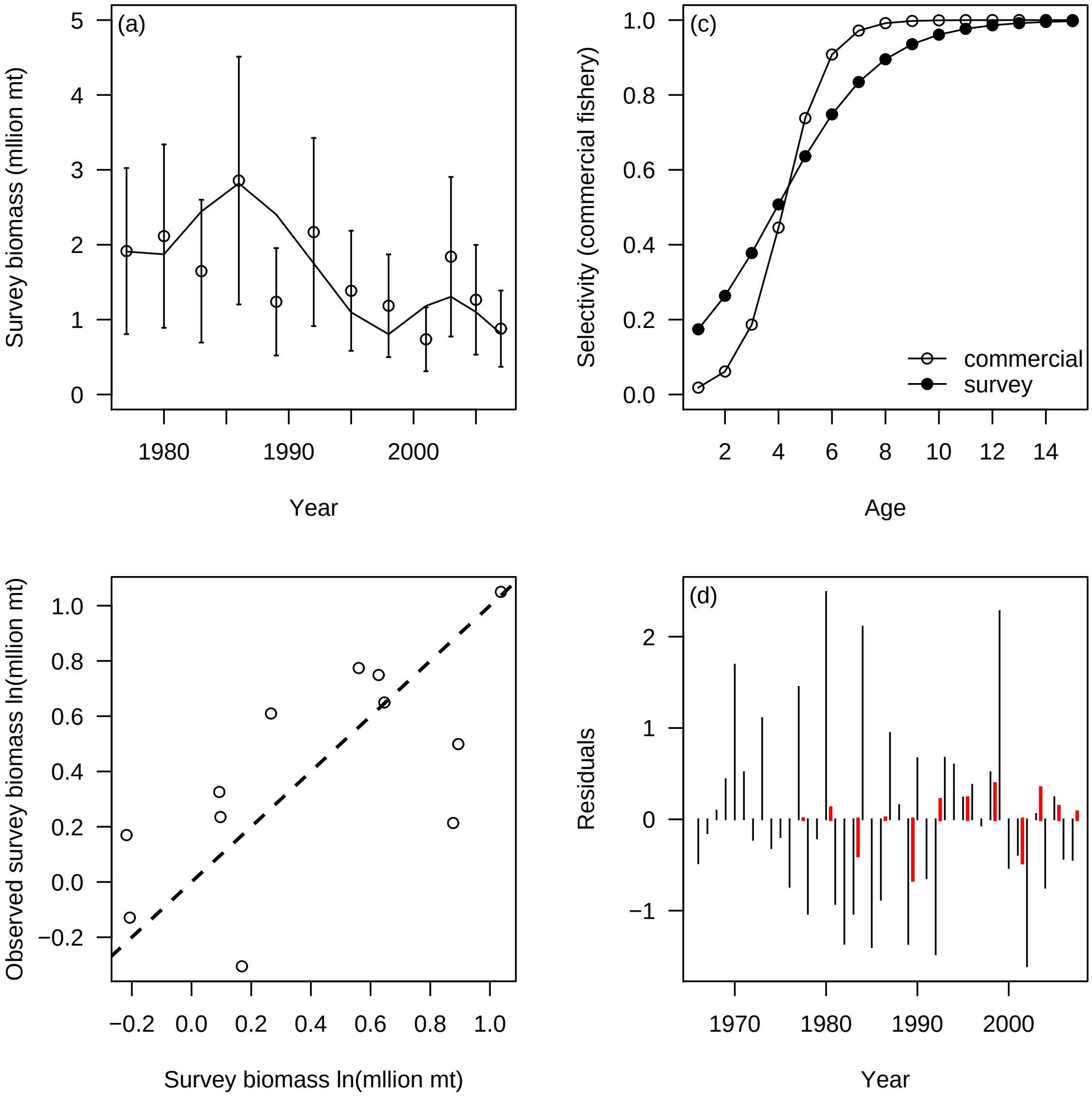

Figure 7: Predicted and observed survey biomass estimates (panel a-b, 1:1 line shown in panel b) based on the maximum likelihood fit to the data. Approximate 95% confidence intervals are shown for the survey points in panel (a) based on the estimated standard deviation in the survey. The estimated selectivity curves for commercial and survey selectivity (c), and the residuals between abundance indices (thick bars in panel d) and recruitemnt deviations (thin bars in panel d).

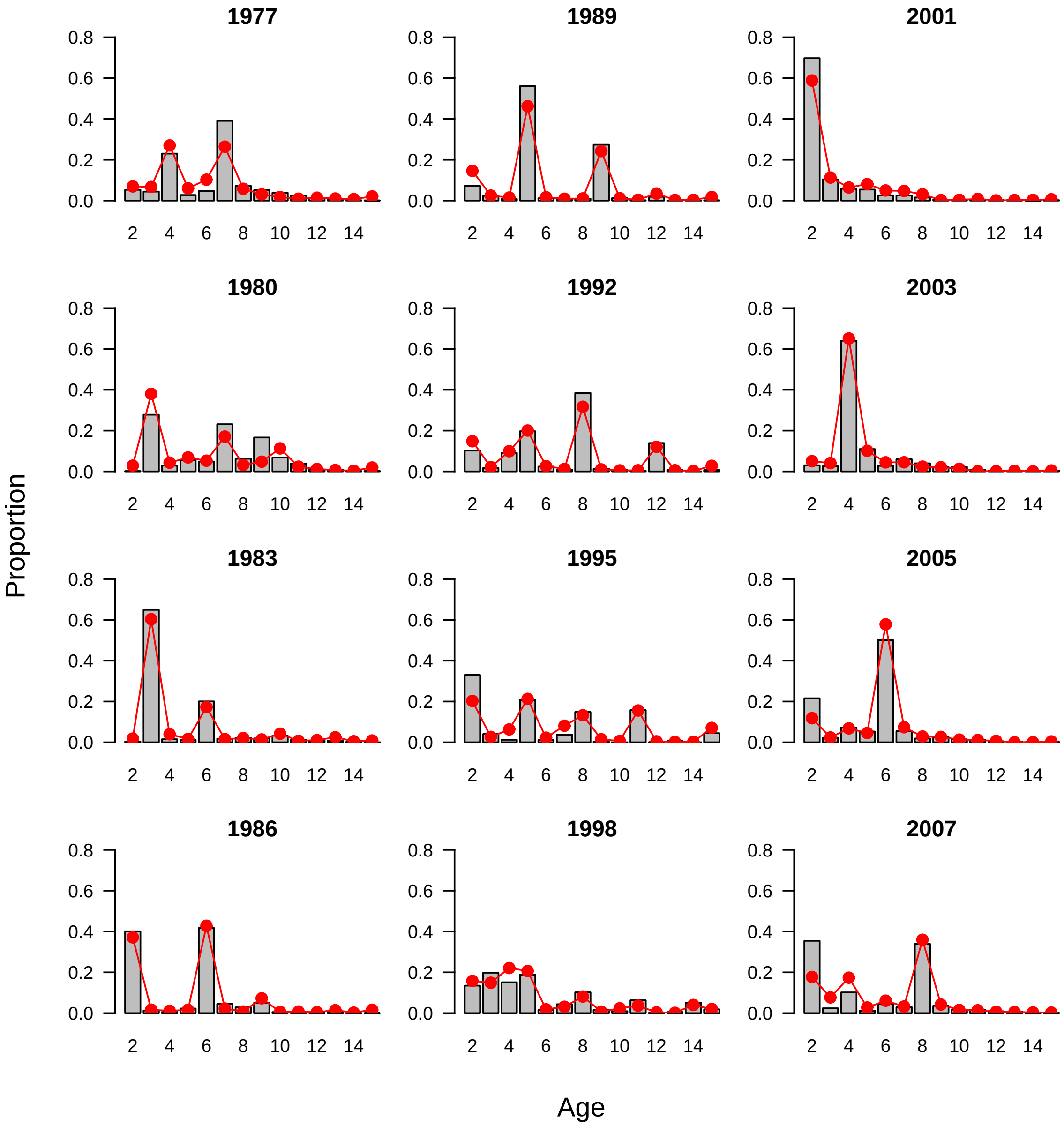

Figure 8: Observed (bars) and predicted (lines) proportions-at-age in the acoustic trawl surveys.

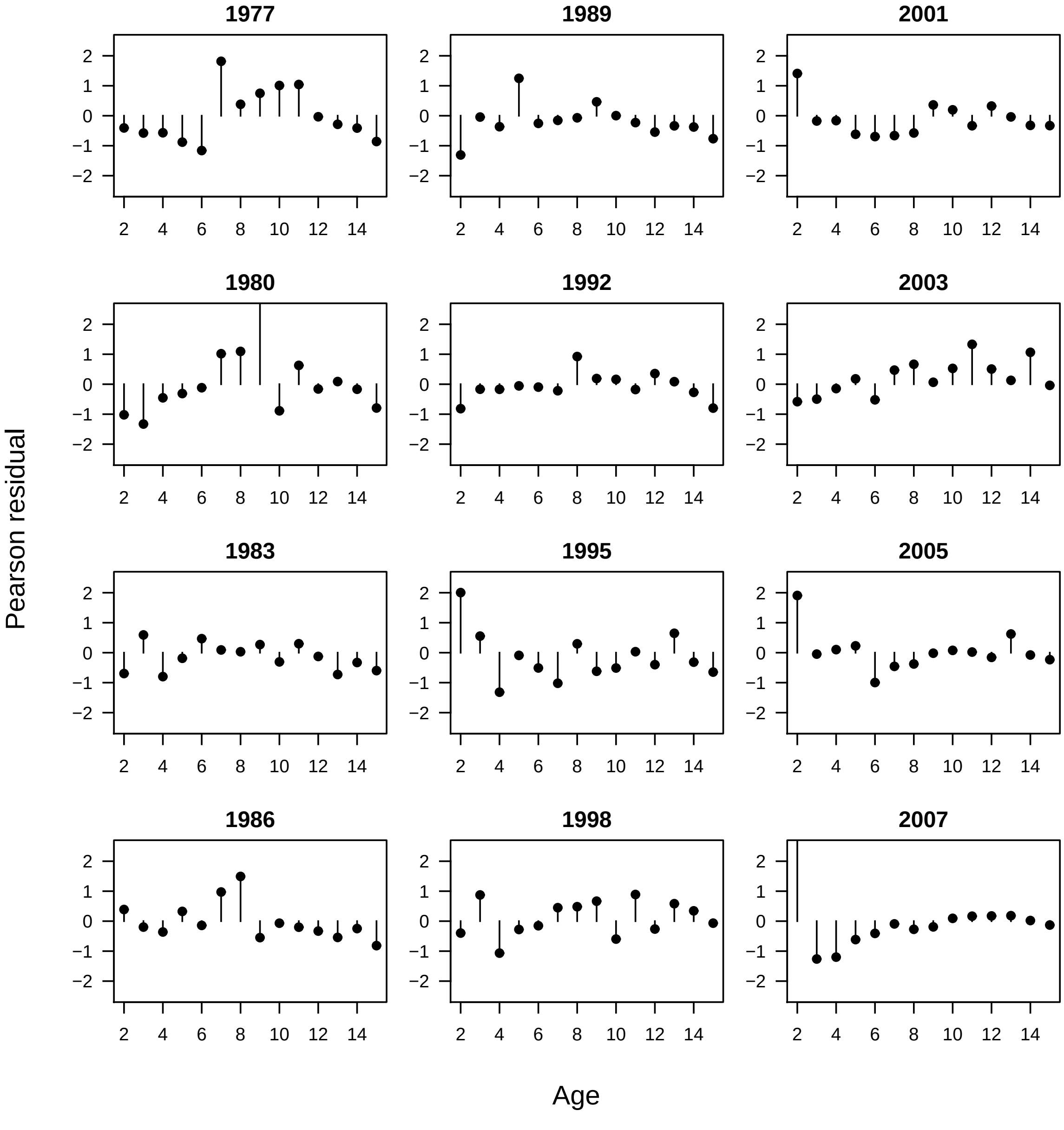

Figure 9: Pearson residuals for the proportions-at-age in the acoustic trawl surveys.

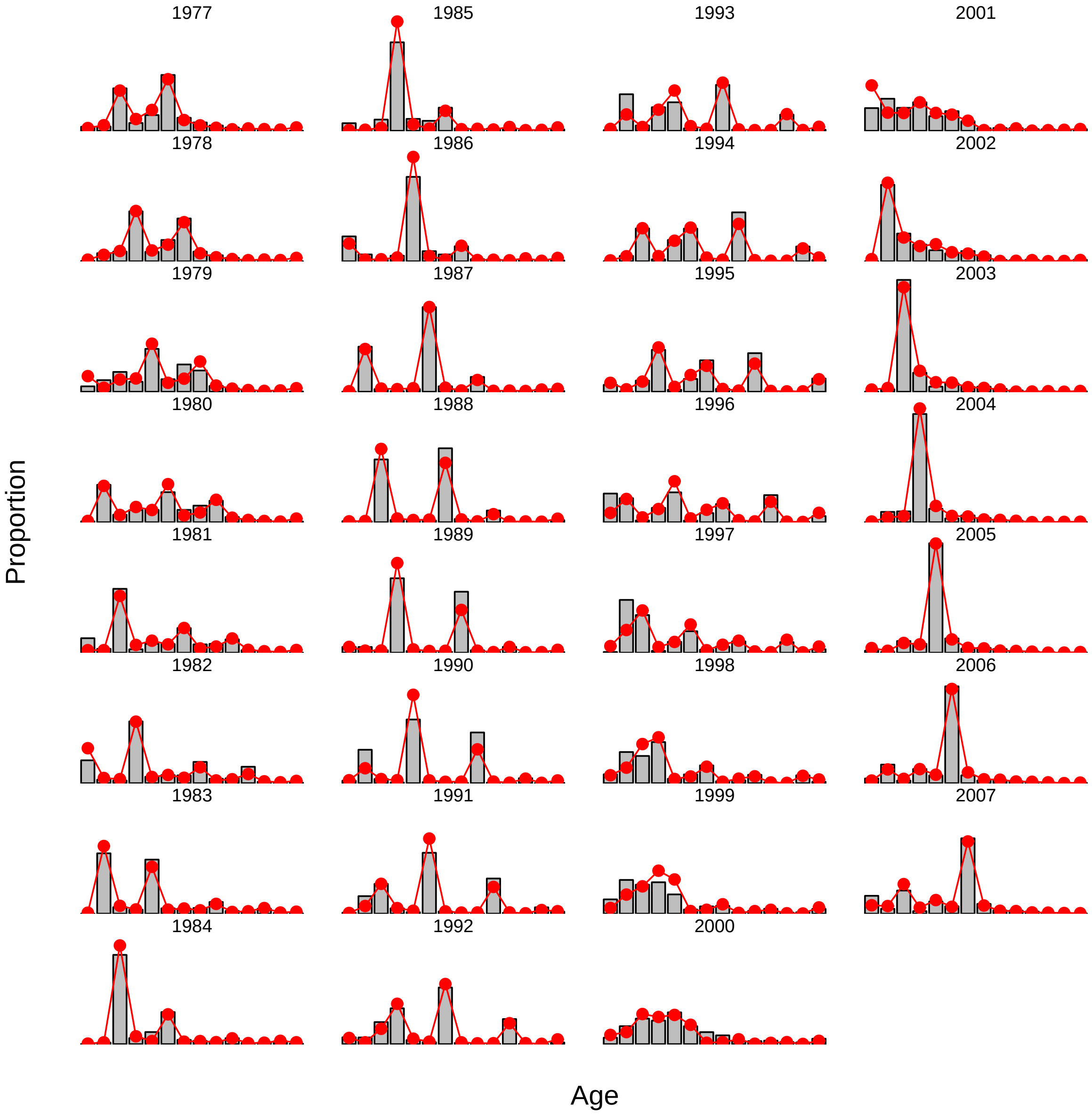

Figure 10: Observed (bars) and predicted (lines) proportions-at-age in the commercial age compositions.

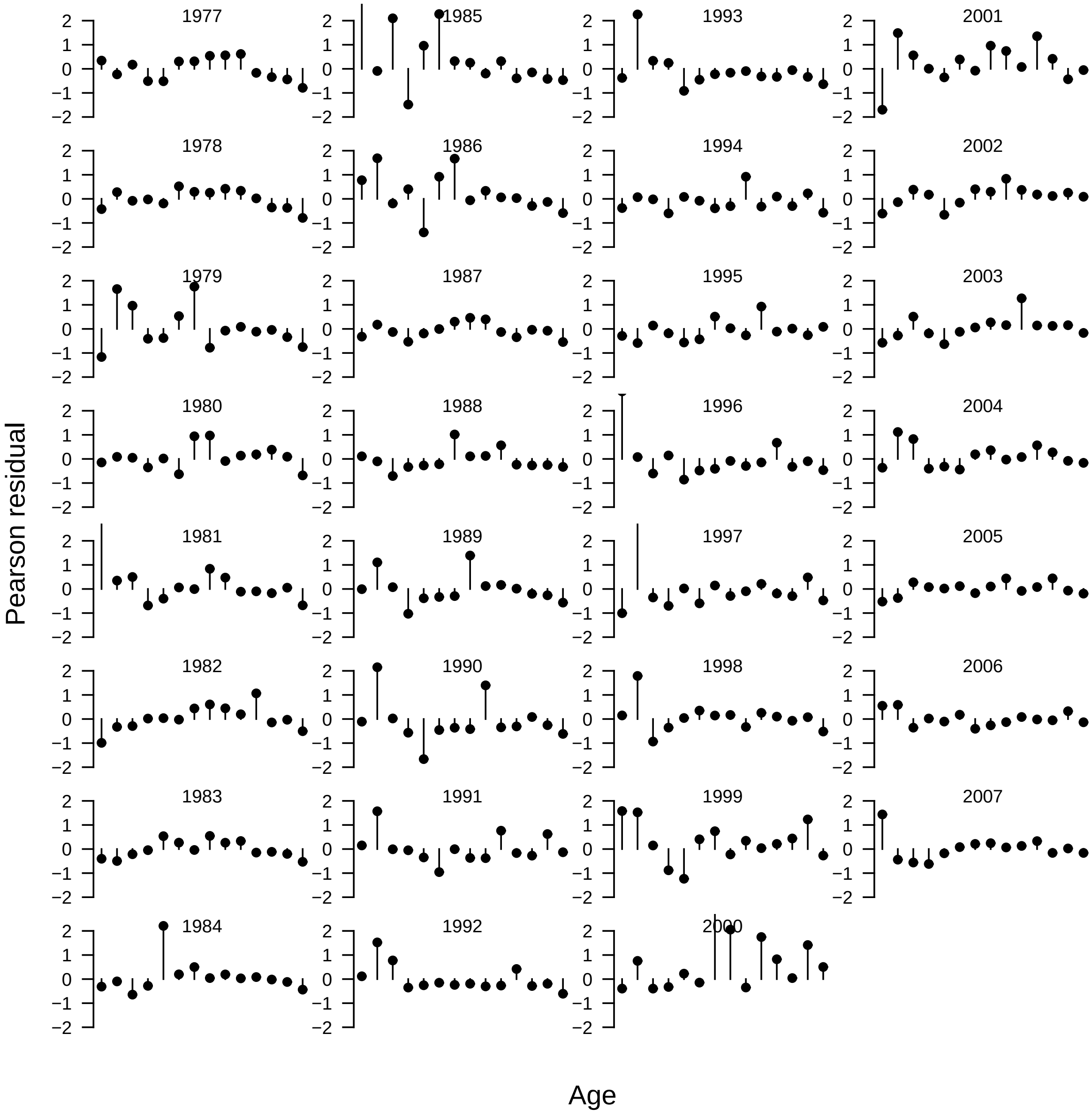

Figure 11: Pearson residuals for the proportions-at-age in the commercial age compositions.

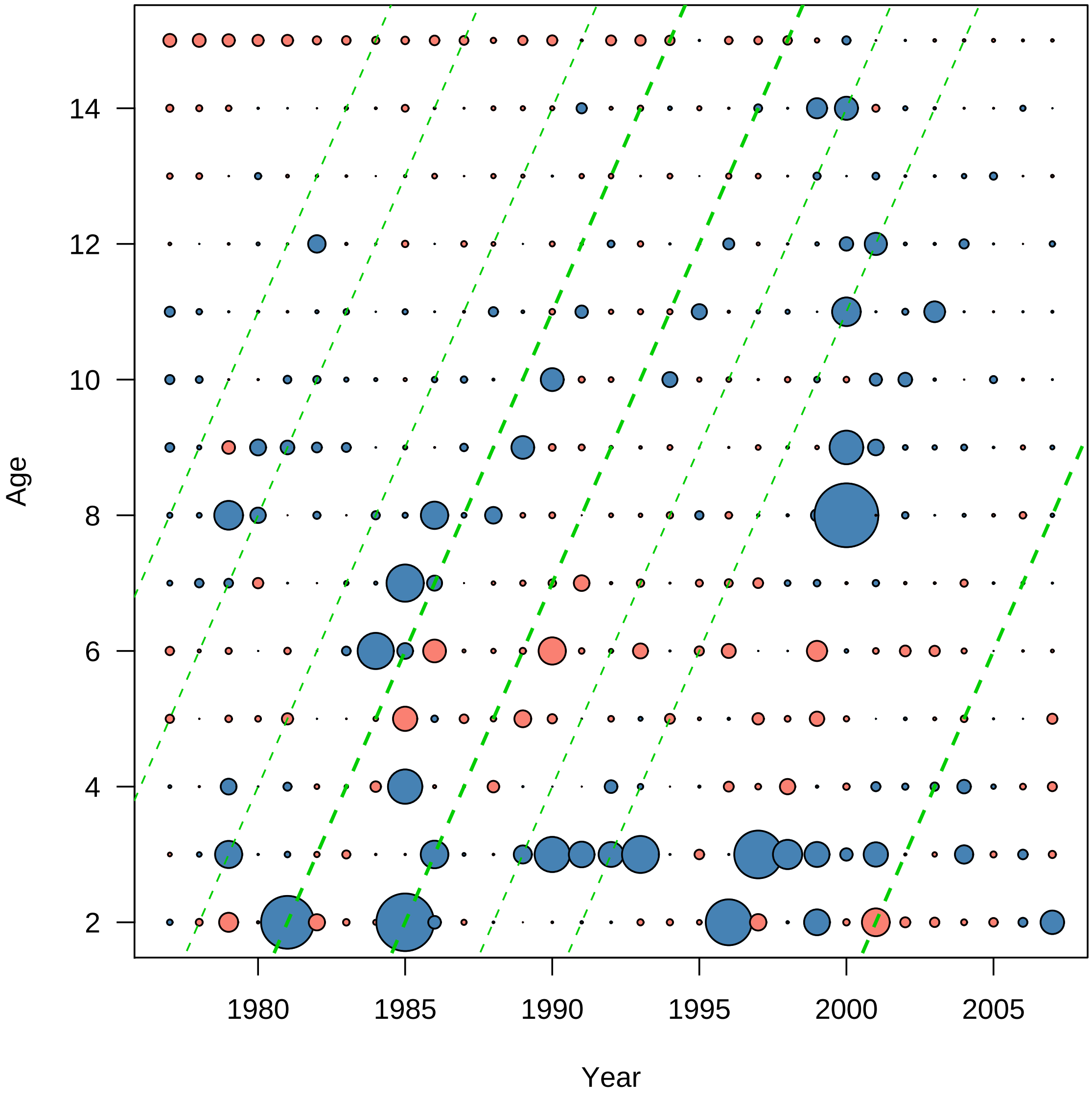

Figure 12: Bubble plots of pearson residuals for the proportions-at-age in the commercial age compositions. Dashed lines follow above average cohorts and the 1980, 1984 and 1999 cohorts are shown in bold dashed lines, positive residuals shown in blue, negative residuals shown in red.

3.1 Results from posterior integration

As reported in Martell et al. (in press), there is insufficient trend information, and an apparent contradiction between the age-composition and trend information to reliably estimate overall population scale and productivity parameters (in this case $C^*$and F^*, and in previous assessments B_o and h). The relative abundance indices are relatively flat, with a slight downward trend between 1986 and 2007. Such one-way trip information is insufficient to resolve parameter confounding between B_o and h, yet this information can be surprisingly informative about MSY (Walters and Martell, 2004).

The marginal distribution for F^*reflects the assumed prior information for F^*(Fig. 13). The median estimate for $C^*$is 0.319 million mt (Table 5), which is greater than the assumed prior mean of 0.2 million mt. Median estimates of M=0.289 are also higher than the assumed prior mean of 0.23 (Table 5). Information to estimate M comes from the age-composition information and is positively correlated with the age at 50% vulnerability parameters ($\hat{a}$ and $\bar{a}$) in the selectivity curves. Note that if a dome-shaped selectivity curve was assumed, then estimates of M would likely decrease owing to the disappearance of older animals due to reduced selectivity. The median estimate of the age at 50% recruitment to the commercial and survey gears is 4.0 and 4.6 years respectively (Table 5). Also, note that the uncertainty in the selectivity parameter is large relative to the commercial selectivity parameters. In particular, the standard deviation in the logistic selectivity curve is sufficiently large that a high proportion of age-2 fish are recruited to the survey gear. The median estimate of the variance ratio ρ is 0.120 and the inverse of the total variance φ^{-2} is 1.139 which corresponds to coefficients of variation of 0.322 and 0.877 for the observation errors and process errors, respectively (Table 5 and Table 7). There is a slight negative correlation between φ^{-2} and C^*(as well as between φ^{-2} and M, Table 7), this illustrates the partial confounding between the age-composition information and the trends in the survey biomass index. As the input sample size for the age-composition is reduced the correlation between φ^{-2} and $C^*$is reduced.

Trends in the median estimates of vulnerable biomass and spawning stock biomass are exactly the same as the maximum likelihood estimates; however, in absolute terms the median estimates are slightly higher than the maximum likelihood estimates (Fig. 14a). Thus, uncertainty in biomass estimates is not normally distributed. In comparison to Helser and Martell (2007), uncertainty is much greater in this assessment owing to the large amount of uncertainty admitted in the global scaling parameter (C^*) and productivity parameter (F^*). Although the survey catchability coefficient (q) is not directly comparable with the assumed values in Helser and Martell (2007), the range of uncertainty in this assessment is much larger than the two options explored in previous assessments (Table 7).

Trends in historical recruitment are also comparable with Helser and Martell (2007), and the median estimates are slightly higher than the maximum likelihood estimates (Fig. 15). The overall uncertainty in annual recruitment is also proportional to the overall uncertainty in the global scaling as well as uncertainty in the estimates of M. The largest cohorts in the past are the 1980, 1984, and 1999, and the 2005 cohort is estimated to be slightly above the long term median historical recruitment. There is a substantial amount of uncertainty in the estimates of age-1 recruits, and this uncertainty owes to the assumed uncertainty in

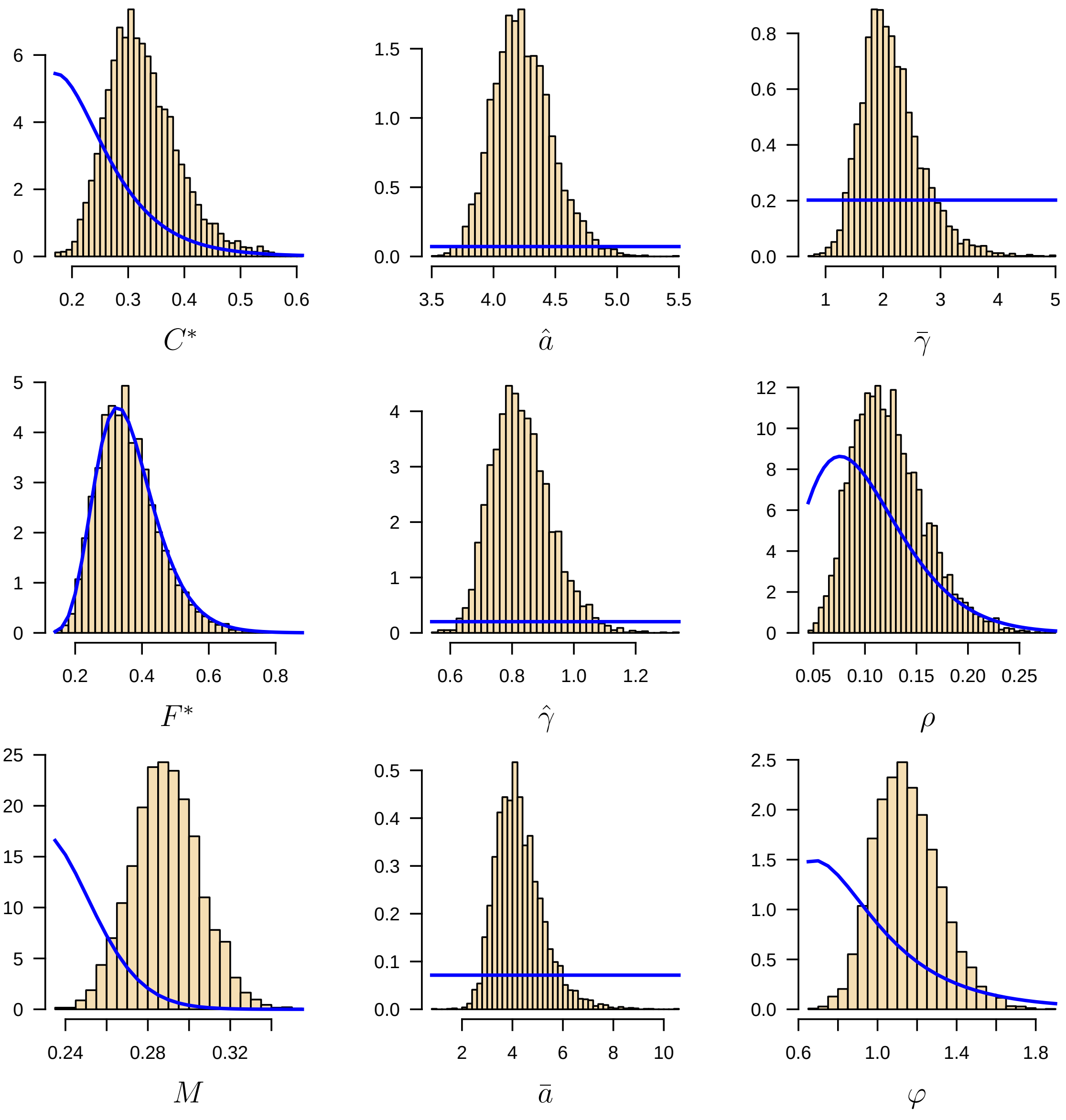

Figure 13: Marginal posterior (histograms) and prior distributions (lines) for key model parameters. Means and variances for the prior distributions are summarized in Table 3.

Table 5: Maximum likeliood estimates (MLE) of model parameters with asymptotic estimates of the standard deviation and median estimates with corresponding 2.5% and 97.5% quantiles from the marginal posterior distributions. Medians and quantiles are based on 5,000 samples from the joint posterior distribution.

	MLE		**Marginal densities**		
	Mean	Std	Median	2.5%	97.5%
C^*	0.305	0.054	0.319	0.224	0.477
F^*	0.330	0.083	0.349	0.215	0.577
M	0.283	0.016	0.289	0.258	0.322
$\hat{a}$	4.174	0.217	4.219	3.812	4.759
$\hat{\gamma}$	0.798	0.081	0.821	0.665	1.038
$\bar{a}$	3.942	0.743	4.154	2.798	6.532
$\bar{\gamma}$	1.890	0.408	2.075	1.311	3.399
ρ	0.119	0.034	0.120	0.067	0.204
φ^{-2}	1.366	0.191	1.139	0.860	1.503

Table 6: Correlation among key model parameters based on 5,000 samples from the posterior distribution.

	C^*	F^*	M	$\hat{a}$	$\hat{\gamma}$	$\bar{a}$	$\bar{\gamma}$	ρ	φ^{-2}
C^*	1.000								
F^*	0.517	1.000							
M	0.504	−0.132	1.000						
$\hat{a}$	−0.108	0.023	0.248	1.000					
$\hat{\gamma}$	−0.124	0.012	0.097	0.857	1.000				
$\bar{a}$	−0.047	0.001	0.225	0.350	0.250	1.000			
$\bar{\gamma}$	−0.099	−0.006	−0.010	0.132	0.099	0.728	1.000		
ρ	−0.043	0.000	0.024	0.015	−0.012	0.027	0.049	1.000	
φ^{-2}	−0.231	0.043	−0.161	0.055	0.056	0.035	−0.001	0.002	1

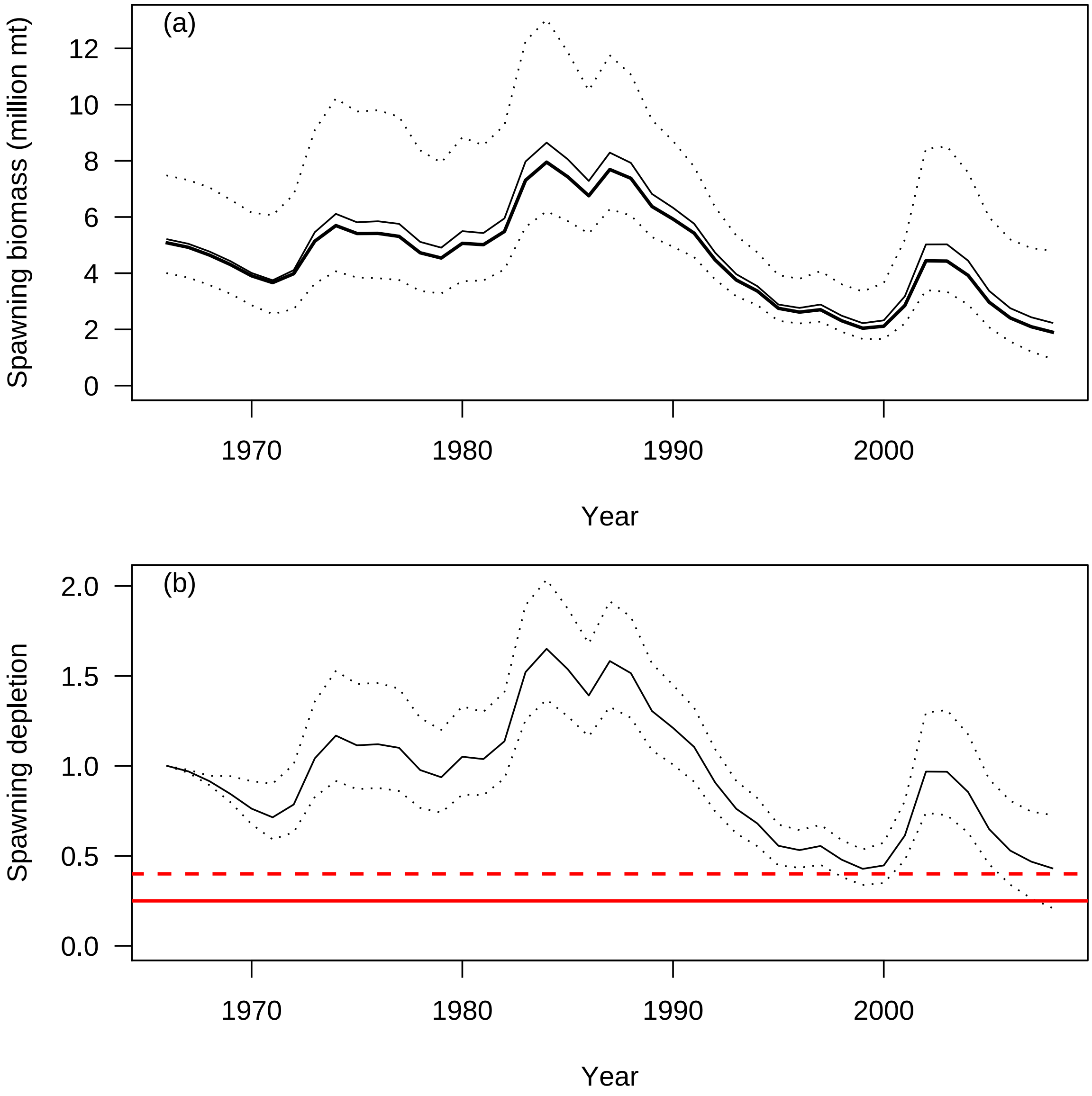

Figure 14: Maximum likelihood estimates (thick line) and median estimates (thin line) of the spawning stock biomass (a) and spawning stock depletion level with 40% and 25% horizontal reference lines (b). The dotted lines represent the 0.025 and 0.975 quantiles based on 5,000 systematic samples from the joint posterior distribution.

Table 7: Modal and median estimates of derived quantities of management interest. Medians and quantiles are based on 5,000 systematic samples from the joint positerior distribution, and the modal estimates correspond to the maximum likelihood estimates.

Derived quantity & Reference piont	Mode	Median	2.5%	97.5%
Survey catchability coefficient (q)	0.477	0.456	0.284	0.723
Steepness (h)	0.488	0.488	0.379	0.627
Spawning stock depletion (2008)	0.375	0.431	0.211	0.727
2008 ABC from 40/10 rule	0.325	0.424	0.072	0.984
Unfished total biomass (B_0)	6.363	6.496	4.856	9.619
Unfished 3+ biomass ($B_{0,3+}$)	5.302	5.44	4.164	7.861
Unfished spawning stock biomass (SB_0)	5.079	5.208	3.999	7.474
Unfished female spawning biomass	2.539	2.604	2	3.737
Spawning stock biomass at MSY (SB_{MSY})	1.839	1.888	1.318	2.883
Female spawning biomass at MSY	0.92	0.944	0.659	1.442
Spawning stock biomass in 2008 (million mt)	1.903	2.235	0.95	4.804
Female spawning stock biomass in 2008 (million mt)	0.951	1.118	0.475	2.402
Coefficient of variation in surveys (σ)	0.295	0.322	0.238	0.44
Coefficeint of variation in recruitment (τ)	0.803	0.877	0.756	1.013

the instantaneous natural mortality rate (M). In comparison to previous assessments the average long-term recruitment is higher; however, both the MLE and median estimates of M are substantially higher than the previously assumed value of 0.23.

Trends in median residual pattern were consistent across all 5,000 samples from the joint posterior distribution (Fig. 16). The 1989 and 2001 acoustic survey biomass estimates are roughly 60% below the predicted biomass. The greatest uncertainty is in the 2007 biomass estimate, and this uncertainty owes to the uncertainty in recent recruitment. The median estimate of the survey catchability coefficient q was 0.456 with a 5% and 95% credible intervals of 0.284 and 0.723, respectively (Table 7). These estimates of q are significantly lower than Helser and Martell (2007); however, in the previous years assessment a dome-shaped selectivity curve for the acoustic survey was assumed and as much as 20% of the older fish were assumed to be "cryptic" biomass.

The median estimate of the spawning stock biomass in 2008 is 2.235 million mt (Table 7) and the modal estimate is 1.903 million mt. More than 20% of 2008 spawning stock biomass it consists of the 1999 cohort (Fig. 17b) and as much as 50% of it consists of the smaller cohorts produced in 2003 and later. Absent any significant recruitment, the spawning stock biomass is expected to decline rapidly as the 1999 cohort ages.

Catch advice based on the 40/10 harvest control rule (ABC in 2008) is highly uncertain, ranging from 72,000 mt to 984,000 mt (Table 7). The modal estimate for the 40/10 rule is 325,000 mt and the median estimate is 424,000 mt. The marginal posterior samples for the

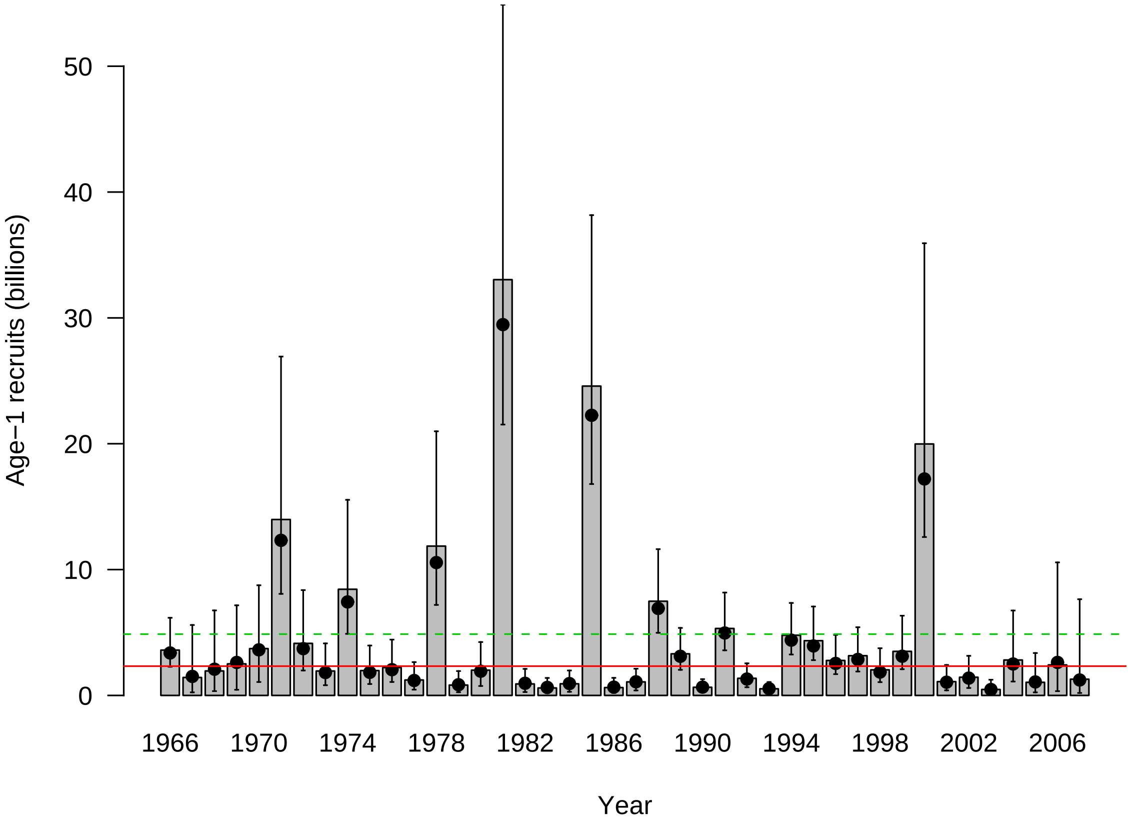

Figure 15: Median (bars) and maximum likelihood (circles) estimates of age-1 recruits, error bars represent the 0.025 and 0.975 quantiles based on 5,000 systematic samples from the joint posterior distribution. Long term average and median recruitment levels are shown as dashed and solid horizontal lines, respectively.

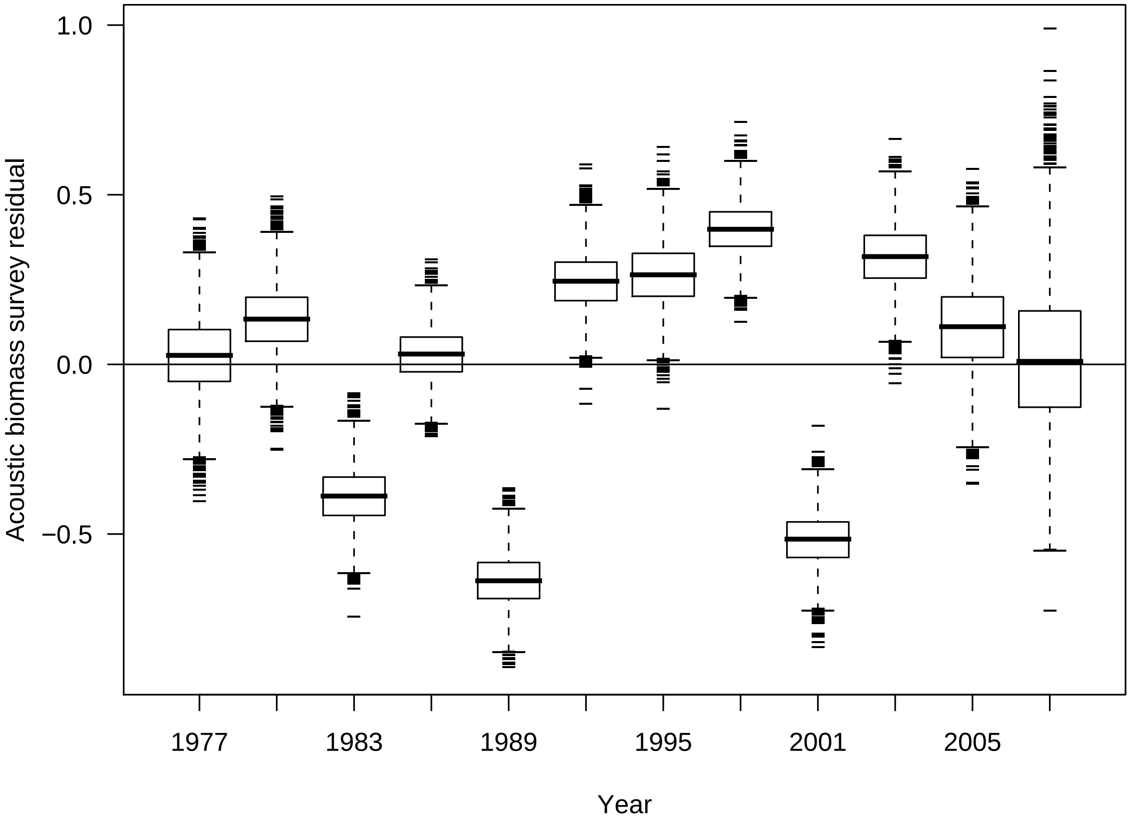

Figure 16: Boxplots of the marginal posteriors for the residuals in the acoustic survey.

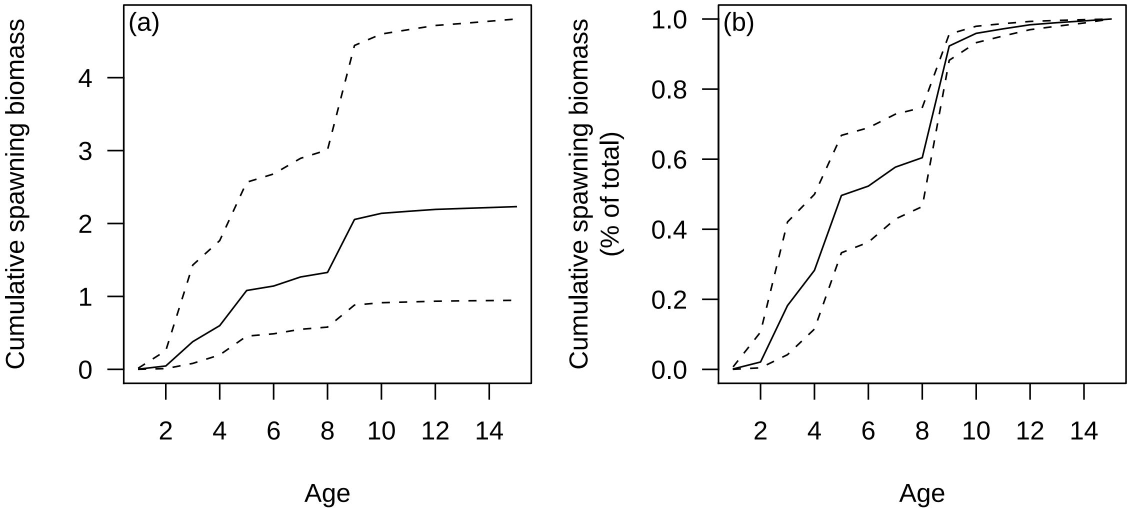

Figure 17: Cumulative spawning stock biomass at-age in 2008. Panel (a) is the cumulative total biomass where the solid line represents the median estimate, and the dashed lines represent the 0.025 and 0.975 quantiles. The cumulative spawning biomass-at-age relative to the total biomass is shown in panel (b).

2008 ABC based on the 40/10 adjustment is highly skewed with a long tail and reflects the huge amount of uncertainty in the 2008 spawning stock biomass estimate.

3.2 Risk analysis

Five different criterion were examined in developing risk profiles for various catch options in 2008. The first criterion is the probability of the fishing mortality rate exceeding the estimated value of F^*(Fig. 18a). First, let 0.25, 0.5 and 0.75 probabilities represent definitions of risk averse, risk neutral, and risk prone, respectively. The risk averse ABC option for the 2008 fishing season based on achieving the target fishing rate of $F^*$is 264,000 mt (Table 8). The risk neutral and risk prone ABC options are 364,000 and 465,000 mt, respectively. The second criterion is the probability of the spawning stock declining between 2008 and 2009 (Fig. 18b). Under this criterion the risk averse to risk prone ABC options are 0, 122,000 and 318,000 mt, respectively (Table 8 column 3). The third criterion examines the probability that the spawning stock biomass in 2009 will fall below the estimate of $\mathrm{SB}_{\mathrm{MSY}}$ (Fig 18c). Under this criterion the probability of the spawning stock falling below $\mathrm{SB}_{\mathrm{MSY}}$ is fairly high with no fishery (P=0.22); the risk neutral and risk prone policies call for ABCs of 464,000 and 920,000 mt (Table 8). The last two criteria criterion examines the probability that the spawning stock will fall below the management target SB_{40} and SB_{25} (Fig 18d). Under

Table 8: Decision table for catch advice. The risk level represents the probability of exceeding a specified management target for a given ABC option. The interpretation of this table is as follows; if the management goal is not to exceed the target fishing mortality rate of F^*in 2008 with a 0.25 probability, then the ABC option should be set at 0.264 million mt or less. If the management target is prevent further decline in spawning stock biomass with a 0.5 probability then the ABC should be set at 0.122 million mt or less.

Risk level	$F_{08} \leq F^*$	$SB_{09} \geq SB_{08}$	$SB_{09} \geq SB_{MSY}$	$SB_{09} \geq SB_{40}$	$SB_{09} \geq SB_{25}$
0.05	0.095	0.000	0.000	0.000	0.008
0.10	0.163	0.000	0.000	0.000	0.225
0.15	0.206	0.000	0.000	0.000	0.360
0.20	0.238	0.000	0.000	0.000	0.462
0.25	0.264	0.000	0.008	0.000	0.546
0.30	0.287	0.000	0.112	0.000	0.619
0.35	0.308	0.011	0.207	0.008	0.685
0.40	0.327	0.049	0.295	0.104	0.748
0.45	0.346	0.086	0.380	0.195	0.807
0.50	0.364	0.122	0.464	0.285	0.866
0.55	0.383	0.157	0.547	0.375	0.924
0.60	0.401	0.194	0.632	0.467	0.984
0.65	0.421	0.232	0.721	0.562	1.046
0.70	0.442	0.273	0.816	0.664	1.113
0.75	0.465	0.318	0.920	0.777	1.186
0.80	0.491	0.370	1.039	0.906	1.270
0.85	0.523	0.432	1.184	1.061	1.371
0.90	0.565	0.515	1.376	1.268	1.506
0.95	0.633	0.649	1.686	1.603	1.724

these criterion, the risk averse policy calls for 0 catch and 456,000 mt for the SB_{40} and SB_{25} policies, respectively.

In summary, catch options in excess of 300,000 mt result in a fairly significant probability of overfishing ($P \geq 0.3$), further declines in spawning stock biomass over present levels, and a significant probability of reducing the spawning stock biomass below SB_{MSY} ($P \geq 0.4$). Catch options less than 300,000 mt result in a very low probability of the spawning stock biomass falling below SB_{25} level ($P \leq 0.15$).

4 Discussion

Uncertainty in previous assessments of Pacific hake was under-represented due to the use of assumed fixed values for the steepness of the stock recruitment relationship and survey

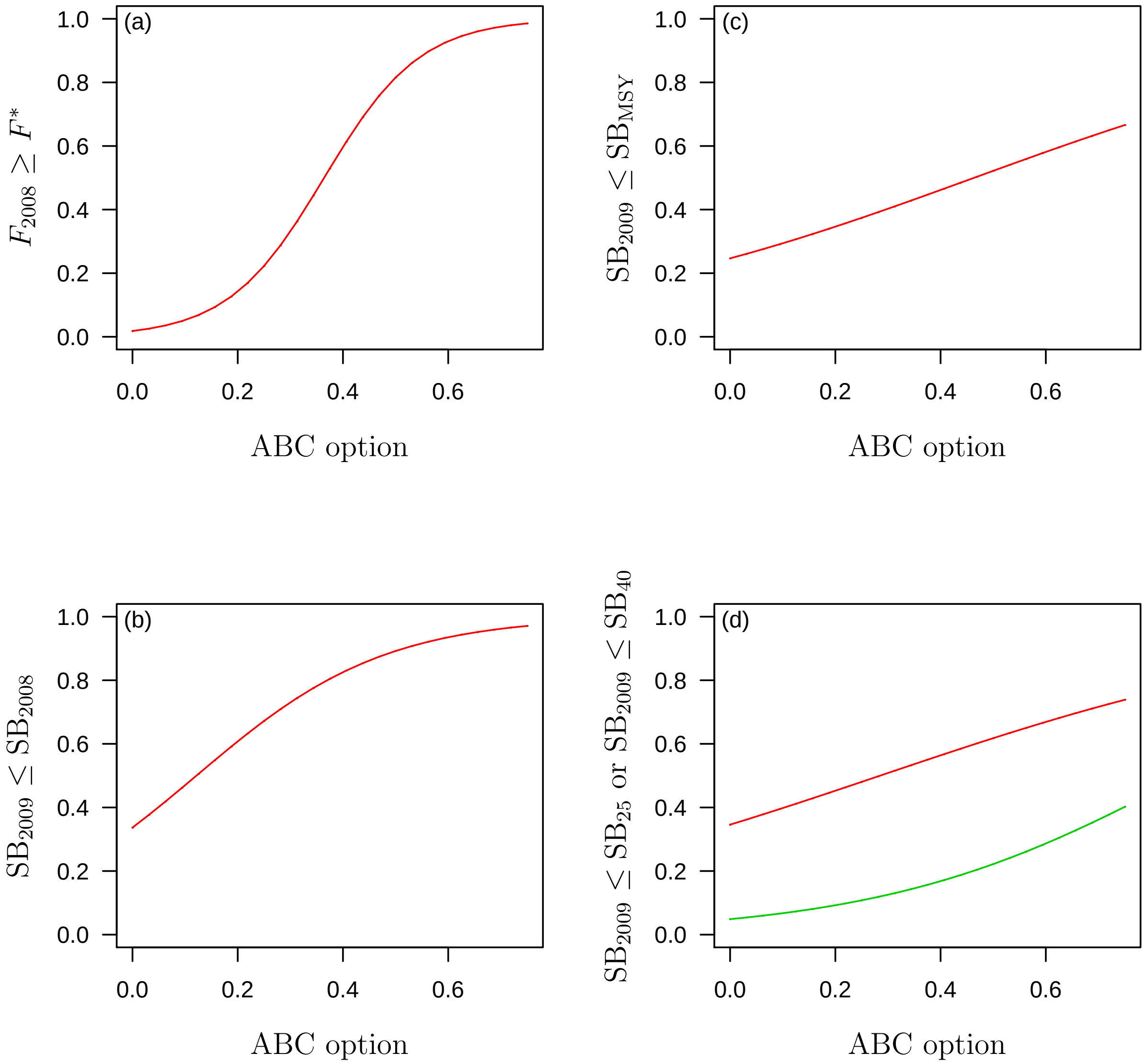

Figure 18: Probability of $F_{2008} > F^*$(panel a) versus ABC option, (b) probability of a decline in spawning biomass ($SB_{2009} < SB_{2008}$) versus ABC option, (c) probability of the SB_{2009} falling below SB_{msy}, and (d) probability of SB_{2009} falling below SB_{25} (bottom line) or SB_{40} (top line) .

catchability coefficients. This assessment attempts to integrate over this uncertainty by using less informative prior information for these key parameters. The relative abundance indices alone lack sufficient information to resolve confounding between the global scaling and stock productivity. Addition of the age-composition information further confounds this problem because there appears to be some conflict between expected trends in abundance due to the exceptional 1980 and 1984 cohorts and the downward trend in abundance between the 1986 and 1989 survey points. Previous assessments have omitted the 1986 survey due to pre- and post-survey calibration problems. However, it appears that the 1986 survey point is consistent with trends inferred from the age-composition data, but the 1989 survey point is inconsistent with these trends.

The biggest source of uncertainty in this assessment lies in the relative weighting of the age-composition information. It is clear that there have been changes in selectivities over time for the commercial gears in the two different countries. Evidence for this is not hard to find; for example, interannual variation in northward migration has profound effects of selectivity. Treating the selectivity curves as constant over time (whether or not a logistic or dome-shaped selectivity curve is assumed) will obviously affect estimates of relative cohort strengths, and down weighting these data tends to reduce the amount of recruitment variation as well as affect age-specific estimates of fishing mortality rates. More importantly, the effect on the catch advice varies greatly over a narrow range of effective sample sizes for the age composition (Table 9).

Table 9: Maximum likelihood estimates of Allowable Biological Catch (ABC million mt), C^*, F^*, steepness (h) and instantaneous natural mortality rates M versus assumed effective sample sizes for the age composition data. Note that these results were generated with the model published in Martell et al. (in press).

Sample N	ABC	C^*(million t)	F^*	h	M
1	0.0526	0.146	0.329	0.374	0.305
5	0.0912	0.172	0.314	0.35	0.332
10	0.142	0.198	0.292	0.366	0.335
33[1]	0.305	0.243	0.271	0.436	0.295
40	0.326	0.246	0.269	0.437	0.293
50	0.648	0.424	0.364	0.663	0.291

There are at least two approaches for dealing with the weighting of the age-compositions: 1) iterative re-weighting as suggested by McAllister and Ianelli (1997), or 2) retrospective analysis (Vivian Haist, pers comm.). We have examined retrospective bias associated with the assumed effective sample sizes for the age-composition information. In summary, the retrospective biased is greatly reduced when effective sample sizes is less than or equal to 15 for both the commercial and survey age-composition information in this analysis. If additional flexibility was incorporated into the model (e.g., dome shaped–time varying selectivity) we would anticipate that the effective sample sizes would increase markedly.

A final point is that the reference points are highly dependent on the assumed maturity-at-age schedule. This information is outdated and given the marked changes in observed growth the maturity at age information should be updated.

5 Acknowledgments

I thank members of the STAT team for discussions about data sources and clarification of how the data was collected, and Ian Stewart for pointing out an error in my likelihood equation. I would also like to acknowledge Carl Walters, Bill Pine, and Jon Schnute for discussions about re-parameterizing models from a management oriented approach.

References

K.M. Bailey, R.C. Francis, P.R. Stevens, National Oceanic, Atmospheric Administration, Northwest, Alaska Fisheries Center (US, and United States. *The Life History and Fishery of Pacific Whiting, Merluccius Productus.* National Oceanic and Atmospheric Administration, National Marine Fisheries Service, Northwest and Alaska Fisheries Center, 1982.

W. G. Clark. Groundfish exploitation rates based on life history parameters. *Canadian Journal of Fisheries and Aquatic Sciences*, 48(5):734–750, 1991.

W. G. Clark. F_{35} Revisited Ten Years Later. *North American Journal of Fisheries Management*, 22(1):251–257, 2002.

R. B. Deriso, M. N. Maunder, and J. R. Skalski. Variance estimation in integrated assessment models and its importance for hypothesis testing. *Can. J. Fish. Aquat. Sci*, 64:187–197, 2007.

MW Dorn and MW Saunders. Status of the coastal Pacific whiting stock in US and Canada in 1997. *Pacific Fishery Management Council, Appendix: Status of the Pacific Coast groundfish fishery through*, 1997.

G.W. Fleischer, K.D. Cooke, P.H. Ressler, R.E. Thomas, S.K. de Blois, L.C. Hufnagle, A.R. Kronlund, J.A. Holmes, and C.D. Wilson. The 2003 integrated acoustic and trawl survey of Pacific hake, *Merluccius productus*, in U.S. and Canadian waters off the Pacific coast. Technical report, U.S. Dept. Commer., NOAA Tech. Memo. NMFS-NWFSC-65, 2005.

RC Francis, National Oceanic, Atmospheric Administration, Northwest, Alaska Fisheries Center (US, and United States. *On the Population and Trophic Dynamics of Pacific Whiting, Merluccius Productus.* National Oceanic and Atmospheric Administration, National Marine Fisheries Service, Northwest and Alaska Fisheries Center, Resource Ecology and Fisheries Management Division, 1982.

C. P. Goodyear. Compensation in fish populations. In C.H. Hocutt and C.H. Jr. Stauffer, editors, *Biological monitoring of fish.* Lexington Books, D.C. Heath Co., Lexington MA., 1980.

T. E. Helser and S. J. D. Martell. Stock Assessment of Pacific Hake (whiting) in U.S. and Canadian Waters in 2007. Technical report, Northwest Fisheries Science Center National Marine Fisheries Service National Oceanic and Atmospheric Administration 2725 Montlake Blvd., East Seattle, WA 98112, USA., 2007.

S. J. D. Martell, W. E. Pine III., and C. J. Walters. Parameterizing age-structured models from a fisheries management perspective. *Can. J. Fish. Aquat. Sci.*, in press.

MK McAllister and JN Ianelli. Bayesian stock assessment using catch-age data and the sampling: importance resampling algorithm. *Canadian journal of fisheries and aquatic sciences(Print)*, 54(2):284–300, 1997.

Otter Research. *An introduction to AD Model Builder for use in nonlinear modeling and statistics*. Otter Research Ltd., Nanaimo, B.C., 1994.

R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2006. URL `http://www.R-project.org`. ISBN 3-900051-07-0.

Jon T. Schnute and Laura J. Richards. Analytical models for fishery reference points. *Can. J. Fish. Aquat. Sci.*, 55:515–528, 1998.

C J. Walters and S. J. D. Martell. *Fisheries Ecology and Management.* Princeton University Press, Princeton, NJ, 2004.

A Input data

Table 10: Combined historical landings (mt) for the U.S. and Can. fisheries, mean age of the catch, and survey abundance indices (millions mt) from the acoustic-trawl survey.

Year	C_t	$\bar{a}$	I_t	Year	C_t	$\bar{a}$	I_t
1966	137700			1987	234147	6.4	
1967	214375			1988	248804	6.7	
1968	122180			1989	305916	6.8	1.238
1969	180131			1990	259792	6.8	
1970	234584			1991	307258	7.3	
1971	154612			1992	296910	7.0	2.169
1972	117546			1993	199435	6.9	
1973	162639			1994	361529	7.8	
1974	211259			1995	249770	7.9	1.385
1975	221360			1996	306075	6.7	
1976	237521			1997	325215	6.0	
1977	132693	6.3	1.915	1998	320619	5.8	1.185
1978	103639	6.6		1999	311855	5.4	
1979	137115	6.6		2000	230820	6.2	
1980	89936	6.9	2.115	2001	235962	5.3	0.737
1981	139121	6.3		2002	182911	5.0	
1982	107734	6.6		2003	205582	5.0	1.840
1983	113924	5.7	1.647	2004	334672	5.4	
1984	138441	5.8		2005	349571	6.2	1.265
1985	110401	5.9		2006	367737	6.5	
1986	210617	6.0	2.857	2007	284358	6.7	0.879

Table 11: Age-composition (reported in percentages) of the combined U.S. and Can. commercial catch from 1977-2007. Age-15 represents a plus group.

Year	age.2	age.3	age.4	age.5	age.6	age.7	age.8	age.9	age.10	age.11	age.12	age.13	age.14	age.15
1977	2.50	2.69	26.05	4.82	9.66	34.25	8.02	5.13	3.26	2.03	1.11	0.41	0.05	0.03
1978	0.24	4.97	5.95	30.66	5.78	13.07	26.25	5.94	3.69	2.10	0.73	0.42	0.18	0.03
1979	3.33	7.13	12.15	6.13	26.31	7.71	16.73	13.04	3.55	2.13	0.85	0.49	0.28	0.19
1980	0.55	22.96	4.64	7.48	7.58	18.46	7.57	10.17	13.21	3.22	1.83	1.41	0.50	0.40
1981	8.90	2.38	39.16	2.11	5.45	5.36	15.13	5.11	5.44	8.17	1.55	0.60	0.57	0.09
1982	14.03	2.14	1.67	37.90	3.92	4.96	4.78	13.05	2.75	2.99	10.08	0.87	0.54	0.32
1983	0.03	37.00	3.97	2.46	33.15	3.26	3.06	3.48	7.15	1.66	1.17	3.06	0.40	0.16
1984	0.00	0.93	54.74	3.71	7.42	19.67	2.63	2.00	1.62	3.67	0.74	0.85	1.73	0.29
1985	4.66	0.54	6.89	54.27	7.32	6.13	14.19	1.46	0.85	1.33	1.26	0.23	0.00	0.87
1986	15.27	4.19	0.87	3.44	51.77	6.22	4.18	9.22	1.31	1.08	0.67	1.09	0.18	0.53
1987	0.00	27.64	1.64	0.39	1.68	51.92	3.36	1.56	9.10	0.40	0.19	0.43	1.21	0.48
1988	0.69	0.59	38.51	1.39	0.80	1.11	45.35	1.99	0.72	7.20	0.13	0.16	0.06	1.30
1989	3.54	3.53	1.52	45.65	1.06	0.44	0.60	37.41	1.49	0.59	3.60	0.09	0.07	0.42
1990	1.55	20.50	2.59	0.44	39.05	0.65	0.25	0.20	31.10	0.36	0.00	3.06	0.01	0.24
1991	0.62	10.75	18.24	3.25	0.96	37.33	1.40	0.13	0.15	21.55	0.51	0.00	3.89	1.22
1992	4.21	4.10	13.53	21.97	2.51	1.18	34.73	0.74	0.13	0.21	15.40	0.20	0.04	1.05
1993	0.43	22.43	3.25	14.46	17.50	1.50	0.79	28.17	0.72	0.05	0.05	9.93	0.06	0.67
1994	0.04	3.30	20.15	1.23	13.10	20.07	1.21	0.43	30.01	0.20	0.43	0.03	9.06	0.73
1995	4.26	0.20	6.86	25.72	1.25	7.88	19.30	1.79	0.31	23.66	0.37	0.26	0.02	8.11
1996	17.60	14.76	1.09	8.82	18.33	1.03	5.65	11.15	0.66	0.34	16.63	0.01	0.11	3.81
1997	0.44	32.38	23.09	1.13	6.71	13.17	1.86	3.73	6.94	1.14	0.14	6.49	0.66	2.12
1998	5.46	19.11	16.70	25.22	2.71	5.39	10.92	1.17	1.79	5.15	0.58	0.13	4.82	0.87
1999	8.76	20.68	17.76	19.25	11.80	2.53	4.45	4.81	0.94	1.66	2.97	0.66	0.85	2.90
2000	3.99	11.01	15.66	14.45	19.49	11.00	7.38	5.39	1.89	1.87	2.16	1.31	1.07	3.34
2001	13.94	19.67	14.09	17.52	8.97	12.05	5.85	1.59	1.71	1.69	1.03	0.90	0.00	1.01
2002	0.05	46.94	17.03	10.23	6.78	4.90	6.29	3.80	0.89	0.66	0.98	0.11	0.40	0.95
2003	0.14	1.55	68.57	11.69	3.13	5.11	3.00	3.11	1.78	0.83	0.24	0.48	0.08	0.28
2004	0.00	6.34	6.67	66.37	8.25	2.32	4.17	2.60	1.39	1.02	0.32	0.24	0.15	0.15
2005	1.26	0.46	7.17	5.43	67.11	8.75	2.46	2.98	2.23	0.94	0.78	0.22	0.03	0.18
2006	2.90	11.42	1.62	8.73	4.78	59.40	4.81	1.67	1.80	1.20	0.84	0.46	0.15	0.22
2007	10.95	2.98	14.19	1.56	7.44	4.53	46.22	5.97	1.96	1.89	1.31	0.42	0.42	0.15

Table 12: Age-composition (percent) from acoustic surveys from 1977-2007. Note that these data are the conditional age-length data multiplied by the length frequencies and collapsed over the size intervals and represent a summary of the conditional age-length data (age 15 represents a plus group).

Year	age.2	age.3	age.4	age.5	age.6	age.7	age.8	age.9	age.10	age.11	age.12	age.13	age.14	age.15
1977	5.31	4.41	23.03	2.71	4.68	39.08	7.21	5.10	3.84	2.45	1.35	0.55	0.17	0.11
1980	0.16	27.80	2.84	5.60	4.84	23.14	6.23	16.63	6.84	3.84	0.92	0.78	0.18	0.20
1983	0.36	64.90	1.50	1.25	20.05	1.75	2.17	1.92	3.25	1.15	0.87	0.70	0.14	0.00
1986	40.10	1.29	0.54	2.28	41.70	4.55	2.85	5.02	0.52	0.49	0.13	0.43	0.06	0.02
1989	7.25	2.35	0.79	56.08	1.15	0.67	0.94	27.39	1.18	0.16	1.87	0.00	0.00	0.17
1992	10.21	1.73	9.12	19.69	2.37	0.86	38.46	1.29	0.67	0.34	13.89	0.67	0.00	0.71
1995	33.02	4.07	1.25	20.71	1.08	3.73	14.85	0.31	0.00	15.78	0.04	0.72	0.00	4.46
1998	13.50	19.82	15.12	18.89	1.54	4.37	10.21	1.64	0.94	6.31	0.14	0.55	5.08	1.89
2001	69.78	10.41	5.79	5.42	2.57	2.49	1.52	0.50	0.52	0.34	0.21	0.20	0.05	0.21
2003	3.01	2.53	64.05	10.95	2.75	6.01	3.96	2.20	2.23	0.73	0.43	0.44	0.31	0.42
2005	21.57	2.27	7.24	5.30	50.03	5.49	1.86	2.61	1.48	1.17	0.49	0.27	0.04	0.19
2007	35.45	2.39	10.19	1.18	4.57	3.01	33.88	3.62	1.74	1.71	0.92	0.80	0.37	0.17

Table 13: Assumed mean weights-at-age in the commercial catch. Note that the mean weight at age for 2007 was based on the mean weights from the previous 5 years.

Year	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1966	0.258	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1967	0.258	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1968	0.259	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1969	0.258	0.429	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.154	1.212	1.269	1.591
1970	0.256	0.428	0.527	0.606	0.680	0.763	0.837	0.935	0.989	1.079	1.155	1.213	1.269	1.589
1971	0.261	0.428	0.527	0.606	0.682	0.762	0.838	0.936	0.988	1.079	1.156	1.213	1.269	1.591
1972	0.256	0.431	0.527	0.606	0.680	0.761	0.837	0.935	0.987	1.077	1.153	1.211	1.267	1.592
1973	0.251	0.423	0.526	0.606	0.680	0.765	0.836	0.935	0.991	1.081	1.155	1.214	1.270	1.582
1974	0.277	0.431	0.528	0.606	0.685	0.760	0.841	0.937	0.987	1.079	1.159	1.215	1.271	1.600
1975	0.241	0.438	0.527	0.605	0.676	0.759	0.833	0.932	0.983	1.073	1.145	1.204	1.261	1.593
1976	0.235	0.400	0.524	0.608	0.679	0.775	0.835	0.936	1.002	1.093	1.162	1.223	1.277	1.554
1977	0.354	0.455	0.533	0.605	0.700	0.748	0.853	0.944	0.974	1.070	1.168	1.218	1.275	1.653
1978	0.135	0.460	0.523	0.600	0.649	0.754	0.812	0.915	0.973	1.055	1.106	1.170	1.231	1.573
1979	0.217	0.287	0.515	0.619	0.686	0.822	0.841	0.951	1.060	1.154	1.211	1.282	1.327	1.435
1980	0.279	0.407	0.487	0.624	0.684	0.796	0.850	0.877	1.010	1.066	1.184	1.163	1.233	1.196
1981	0.123	0.328	0.491	0.619	0.725	0.776	0.816	0.864	0.884	1.043	1.189	1.245	1.213	1.385
1982	0.235	0.389	0.503	0.604	0.688	0.839	0.873	0.907	0.934	1.029	1.049	1.132	1.209	1.095
1983	0.264	0.355	0.428	0.563	0.631	0.742	0.827	0.855	0.883	0.969	0.994	0.941	1.155	1.095
1984	0.215	0.393	0.429	0.531	0.669	0.699	0.796	0.873	0.894	0.953	1.104	0.965	1.008	1.100
1985	0.181	0.316	0.455	0.526	0.639	0.740	0.813	0.979	0.914	1.020	1.035	1.156	1.074	1.067
1986	0.273	0.314	0.426	0.537	0.562	0.633	0.724	0.821	0.921	0.992	0.989	1.102	1.048	1.086
1987	0.232	0.374	0.421	0.499	0.629	0.626	0.683	0.746	0.799	0.903	0.895	1.023	0.950	1.049
1988	0.264	0.357	0.443	0.461	0.598	0.591	0.628	0.687	0.775	0.809	0.895	0.998	0.993	1.026
1989	0.226	0.317	0.367	0.502	0.531	0.617	0.656	0.670	0.717	0.789	0.896	0.860	1.052	1.030
1990	0.272	0.379	0.443	0.531	0.568	0.617	0.604	0.604	0.701	0.749	0.822	0.880	1.002	1.052
1991	0.229	0.341	0.449	0.543	0.554	0.641	0.716	0.599	0.885	0.728	0.724	0.854	0.952	1.060
1992	0.248	0.338	0.458	0.525	0.582	0.598	0.638	0.638	0.612	0.679	0.698	0.851	0.716	0.931
1993	0.263	0.343	0.426	0.502	0.560	0.593	0.547	0.638	0.645	0.704	0.931	0.679	0.798	0.756
1994	0.335	0.344	0.424	0.510	0.552	0.608	0.694	0.620	0.689	0.636	0.739	0.812	0.725	0.794
1995	0.114	0.515	0.484	0.511	0.625	0.623	0.679	0.706	0.713	0.724	0.661	0.892	0.711	0.771
1996	0.271	0.379	0.462	0.547	0.565	0.628	0.621	0.663	0.712	0.736	0.705	0.553	1.092	0.724
1997	0.328	0.409	0.472	0.519	0.615	0.620	0.601	0.692	0.665	0.741	0.732	0.743	0.696	0.813
1998	0.234	0.350	0.458	0.497	0.518	0.587	0.598	0.619	0.637	0.651	0.775	0.638	0.735	0.734
1999	0.243	0.318	0.417	0.538	0.554	0.578	0.625	0.661	0.672	0.748	0.727	0.746	0.661	0.786
2000	0.282	0.424	0.496	0.564	0.647	0.677	0.658	0.740	0.719	0.818	0.746	0.835	0.786	0.820
2001	0.289	0.454	0.599	0.608	0.681	0.778	0.780	0.806	0.854	0.832	0.831	0.901	0.863	0.962
2002	0.310	0.413	0.558	0.752	0.702	0.812	0.916	0.885	0.885	0.927	0.893	1.064	1.002	1.100
2003	0.304	0.380	0.469	0.573	0.664	0.659	0.679	0.732	0.709	0.766	0.752	0.709	0.827	0.941
2004	0.241	0.419	0.489	0.550	0.625	0.709	0.691	0.713	0.757	0.765	0.742	0.880	0.928	0.836
2005	0.333	0.426	0.497	0.550	0.573	0.612	0.647	0.693	0.680	0.729	0.722	0.804	0.629	0.760
2006	0.251	0.418	0.497	0.552	0.584	0.607	0.645	0.785	0.744	0.798	0.838	0.866	0.801	0.805
2007	0.288	0.411	0.502	0.596	0.629	0.680	0.716	0.762	0.755	0.797	0.789	0.865	0.838	0.888

B Model description and documentation

The stock assessment model used herein consists of 4 major components: 1) a component for initializing the model based on steady-state conditions, 2) a component for updating the state variables, 3) a component that relates the state variables to observations on relative abundance and composition information, and 4) a statistical criterion for evaluating how likely these data are for a given set of model parameters. We have broken the description of the assessment model into these four components and use a series of tables to document model equations. Symbols and their definitions are defined in Table 14; furthermore, we have divided the estimated parameter set into life-history parameters Φ and population parameters Θ for clarity.

We have adopted a management oriented approach tho the parameterization of the age-structured model where the leading parameters that define population scale and productivity correspond to MSY (hereafter C^*) and Fmsy (hereafter F^*). The basic idea here is to change the question to how likely are the data given $C^*$and $F^*$and derive the corresponding B_o and slope of the stock recruitment relationship rather than the traditional approach of estimating these values directly. There are a few statistical advantages of using this approach (i.e., reduced confounding between the leading parameters Schnute and Richards, 1998), but perhaps the biggest advantage is to increase the transparency by which the application of informative priors influence model results (Martell et al., in press).

B.1 Model initialization

To initialize the model, we must first derive B_o and κ from $C^*$and F^*as well as other life-history parameters Φ and the vulnerability schedule. In other words, first we must transform the management parameters $C^*$and F^*into population parameters B_o and κ. This transformation starts with the equilibrium yield equation (e.g. Fig 19a), differentiating this function with respect to F_e, setting this equation equal to 0 and solving for κ (for the full derivation see Martell et al., in press). Next substitute κ back into the equilibrium recruitment equation to obtain estimates of the unfished biomass B_o.

An alternative way to envision this transformation is to think about it graphically. For any given model (e.g., a simple production model or a complex age-structure model) we can derive a system of equation that results in the equilibrium yield for any specified equilibrium fishing mortality rate. This same system of equations can also be used to derived equilibrium values of recruitment (e.g., Fig 19b), equilibrium biomass (e.g., Fig 19c) and the spawners per recruit (Fig. 19d). The traditional approach would then differentiate the catch equation with respect to F_e, solve this expression for F_e to determine the corresponding value of F^*, then substitute the corresponding F^*into the catch equation and calculate C^*conditional on estimates of B_o and κ. What differs in the management oriented approach is that we estimate $C^*$and F^*directly and then derive B_o and κ conditional on the estimates of $C^*$and F^*.

The system of equation used to derive B_o and κ are laid out in Table 15. The purpose of

Table 14: Description of symbols and indices used in TINSS

Symbol	Description
Indices	
i, j, k, l	index for age,year, fleet, and size interval
Estimated population parameters (Θ)	
F^*	Optimal fishing mortality rate
C^*	Maximum sustainable yield
M	Instantaneous natural mortality rate
a_{h_k}	Age at 50% selectivity
γ_k	Standard deviation in selectivity
Estimated life-history parameters (Φ)	
l_∞	mean asymptotic length
k	growth coefficient
t_o	age at 0 length
a, b	parameters for length-weight relationship
λ_1, λ_2	parameters for standard deviation in length-at-age
Derived variables	
B_o	unfished steady-state biomass
κ	recruitment compensation ratio (Goodyear, 1980)
R_e	equilibrium age-1 recruitment
$\iota_i, \hat{\iota}_i$	survivorship to age i, unfished and fished
ϕ_E, ϕ_e	eggs per recruit, unfished and fished
ϕ_B, ϕ_b	vulnerable biomass per recruit, unfished and fished
ϕ_q	vulnerable biomass available to the fishery

laying out the equations in a tabular format is two fold, 1) documentation of the model structure and 2) to provide an algorithm or pseudo code in which to implement the model. First given initial estimates of the life-history parameters Φ (T15.2), calculate the corresponding age-schedule information (T15.3)–(T15.6). Note that this does not assume that growth or maturity is constant over time, only that some average, or steady state, growth occurred for the cohorts that are used to initialize the numbers-at-age. Next, calculate the survivorship (T15.7) of an individual recruit based on the instantaneous natural mortality rate M. These survivorship functions (T15.7) and (T15.8) are used to calculate the per recruit incidence functions for unfished and fished conditions, respectively. An incidence function is the sum of age-specific schedules that express the population units on a per recruit basis. For example the total biomass per recruit is given by (T15.10) and the total unfished biomass is the product $R_o\phi_E$. For notational purposes the prefix ϕ denotes an incidence function and the corresponding subscript denotes the type of incidence function (see Table 14 for definitions); we also use upper and lower case subscripts to denote unfished and fished conditions, respectively.

The eggs per recruit for unfished and fished conditions are defined by (T15.9), the biomass

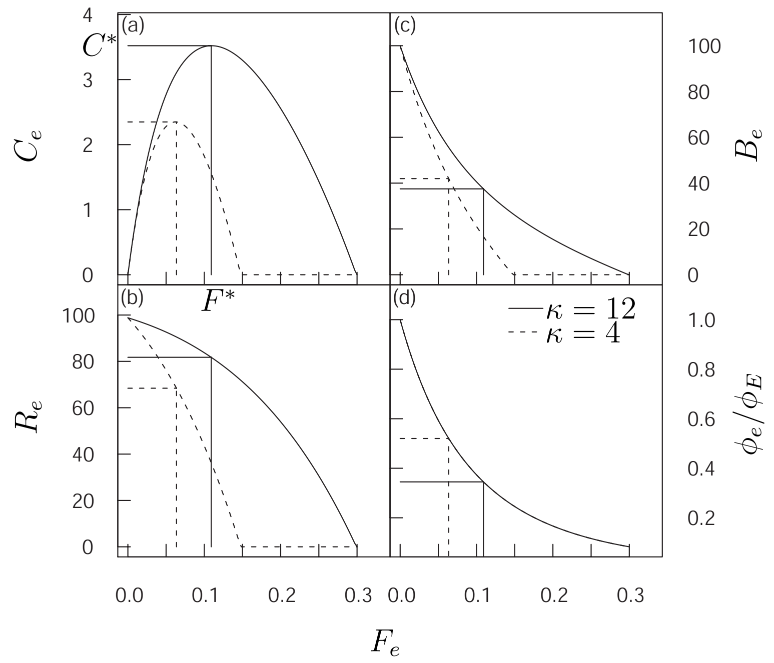

Figure 19: Relationship between equilibrium values for yield (a), recruitment (b), biomass (c) and spawners per recruit (d) versus instantaneous fishing mortality rate for a hypothetical stock with high ($\kappa = 12$) and low ($\kappa = 4$) recruitment compensation parameters.

per recruit by (T15.10), and the vulnerable biomass per recruit available to the fishery is defined by (T15.11). Note that we assume both natural and fishing mortality operate simultaneously and ϕ_q represents the Barnov catch equation. To derive κ, we differentiate

$$C_e = F_e R_e \phi_q \tag{1}$$

with respect to F_e and solve this equation for κ. Using the chain rule, the derivative of (1) is

$$\frac{\partial C_e}{\partial F_e} = R_e\phi_q + F_e\phi_q\frac{\partial R_e}{\partial F_e} + F_eR_e\frac{\partial \phi_q}{\partial F_e} \tag{2}$$

To derive the recruitment compensation parameter (T15.12) it is necessary to substitute (T15.11) and (T15.13) into (2), set the corresponding expression equal to zero and then solve for κ. The partial derivatives for (T15.12) are defined in Table 16. Equation (T15.13) is the equilibrium recruits that corresponds to the equilibrium fishing mortality rate F_e and (T15.14) corresponds to the unfished biomass.

B.1.1 Initialization with multiple fleets

Although the catch data are aggregated into a single fleet for this assessment, the following describes an algorithm for implementing the management oriented approach for multiple fleets that have different age-specific fishing mortality rates. In essence, the algorithm derives F-multipliers for each fleet.

The catch equation (1) considers a single fishery with a unique vulnerability-at-age curve. In the case of multiple fisheries with different vulnerability-at-age curves, it is necessary to allocate the proportion of the total fishing mortality (F^*) to each fleet such that the sum of catches from each fleet is equal to C^*. For example, consider two separate fishing fleets A and B and assume that fleet A harvest younger fish that fleet B and that the allocation of $C^*$is assigned equally to each fleet. In this case a higher proportion of F^*would be assigned to fleet B because this fleet harvest fewer, older fish, in comparison to fleet A which harvest more abundant younger fish. Thus, if some sort of allocation agreement exists between two or more fleets, a multiplier on the fishing mortality rate must be used to allocate the total catch among these fleets. For a given allocation arrangement (e.g., where the fraction of C^*assigned to fleet k is denoted as Λ_k), the equilibrium catch of fleet k can be represented as:

$$\Lambda_k C^* = \tau_k F^* R_e \phi_q^{(k)} \tag{3}$$

where τ_k is the fleet specific multiplier on F^*, R_e is defined in (T15.13), and $\phi_q^{(k)}$ is the fleet specific vulnerable biomass per recruit which is defined as

$$\begin{aligned} \phi_q^{(k)} &= \sum_i \frac{\hat{\iota}_i w_i v_{i,k}}{Z_i}\left(1 - e^{-Z_i}\right), \\ \text{where} \quad Z_i &= M + F^* \sum_k \tau_k v_{i,k}, \\ \hat{\iota}_i &= \begin{cases} 1 & i = 1 \\ \hat{\iota}_{i-1} e^{-Z_{i-1}} & i > 1. \end{cases} \end{aligned} \tag{4}$$

Note that τ_k appears multiple times in (4) in the Z_i and $\hat{\iota}_i$ terms, as well as the derivation of R_e (see eq. T15.13), and there is no analytical solution for τ_k (at least that we could find using symbolic math languages). Therefore, τ_k must be solved for iteratively. Solving (3) for τ_k results in an update of τ_k:

$$\tau_k = \frac{\Lambda_k C^*}{R_e F^* \phi_q^{(k)}} \tag{5}$$

A simple algorithm to numerically calculate τ_k proceeds as follows

1. set initial values of the fishing multiplier equal to the allocation proportion: $\tau_k = \Lambda_k$ (Note that if the vulnerability-at-age curves are the same for each fleet, then τ_k is exactly equal to Λ_k, i.e., the vulnerable biomass per recruit is the same for all fleets).

2. calculate the age-specific total mortality rates for all fleets combined

$$Z_i = M + F^* \sum_k \tau_k v_{i,k}.$$

3. calculate survivorship ($\hat{\iota}_i$), and per-recruit incidence functions that lead to R_e (eqs. T15.8–T15.13) based on the age-specific total mortality rate in step 2.

4. for each fleet k, calculate the vulnerable biomass per-recruit ($\phi_q^{(k)}$) using (4).

5. update τ_k using (5), and repeat steps 2-5 until estimates of τ_k converge (Note this take 6-20 iterations depending on how different the vulnerability-at-age curves are for each fleet.

6. Check that the sum catches for each fleet equal C^*.

The algorithm outline above is based on the allocation arrangement among the various fleets (Λ_k) and is not intended to optimize the allocation arrangement based on differences in vulnerability among the various fishing fleets. This is an entirely different policy issue that is not addressed here. If there is no formal allocation arrangement, then historical catch proportions to each fleet could be used as a starting point for values of Λ_k. Recall, that the approach adopted here is to simple express the population parameters B_o and κ as analytical functions of management parameters $C^*$and F^*.

B.2 Updating state variables

Equations used to update the state variables are defined in Table 17. We aggregate the catch data from the CAN and US fisheries into a single catch time series (T17.1) and treat both fisheries as a single fishery with the same selectivity pattern over time. This data simplification reduces the number of estimated parameters but further assumes that the relative mortalities imposed by the two different fisheries has been constant over time. We also aggregate the catch-age samples from the commercial fisheries ($A_{i,j}$) into a single catch age matrix. Catch-age data for the US portion of the fishery are available back to 1976, and age-composition information for the CAN portion of the fishery are available back to 1988. The age-compositions were combined from 1988 to 2006 using a weighted average, where the weights are the proportions landed by each nation. The relative abundance data (I_j) corresponds to the abundance index derived from the acoustic surveys, and here we assume these indices are proportional to abundance and estimate the scaling parameter.

Table 15: Steady-state age-structured model assuming unequal vulnerability-at-age, age-specific natural mortality, age-specific fecundity and Beverton-Holt type recruitment.

Parameters

$$\Theta = (C^*, F^*, M, \hat{a}, \hat{\gamma}); \quad C^* > 0; F^* > 0; M > 0 \tag{T15.1}$$

$$\Phi = (l_\infty, k, t_o, a, b, \dot{a}, \dot{\gamma}) \tag{T15.2}$$

Age-schedule information

$$l_i = l_\infty(1 - \exp(-k(a - t_o))) \tag{T15.3}$$

$$w_i = a(l_i)^b \tag{T15.4}$$

$$v_i = (1 + \exp((\hat{a} - a)/\hat{\gamma}))^{-1} \tag{T15.5}$$

$$f_i = w_i(1 + \exp((\dot{a} - a)/\dot{\gamma}))^{-1} \tag{T15.6}$$

Survivorship

$$\iota_i = \begin{cases} 1, \quad i = 1 \\ \iota_{i-1} e^{-M}, \quad i > 1 \\ \dfrac{\iota_{i-1}}{1 - e^{-M}}, \quad i = A \end{cases} \tag{T15.7}$$

$$\hat{\iota}_i = \begin{cases} 1, \quad i = 1 \\ \hat{\iota}_{i-1} e^{-M - F^* v_{i-1}}, \quad i > 1 \\ \dfrac{\hat{\iota}_{i-1}}{1 - e^{-M - F^* v_i}}, \quad i = A \end{cases} \tag{T15.8}$$

Incidence functions

$$\phi_E = \sum_{i=1}^{\infty} \iota_i f_i, \quad \phi_e = \sum_{i=1}^{\infty} \hat{\iota}_i f_i \tag{T15.9}$$

$$\phi_B = \sum_{i=1}^{\infty} \iota_i w_i, \quad \phi_b = \sum_{i=1}^{\infty} \hat{\iota}_i w_i v_i \tag{T15.10}$$

$$\phi_q = \sum_{i=1}^{\infty} \frac{\hat{\iota}_i w_i v_i}{M + F^* v_i} \left(1 - e^{(-M - F^* v_i)}\right) \tag{T15.11}$$

Derived variables

$$\kappa = \frac{\phi_E}{\phi_e} - \frac{F^* \phi_q \dfrac{\phi_E}{\phi_e^2} \dfrac{\partial \phi_e}{\partial F^*}}{\phi_q + F^* \dfrac{\partial \phi_q}{\partial F^*}} \tag{T15.12}$$

$$R_e = \frac{C^*}{F^* \phi_q} \tag{T15.13}$$

$$B_o = \phi_B \frac{R_e(\kappa - 1)}{\kappa - \phi_E/\phi_e} \tag{T15.14}$$

Table 16: Partial derivatives, based on components in Table 15, required for the derivation of κ and B_o using the Beverton-Holt recruitment model.

Mortality & Survival

$$Z_i = M + F^* v_i \tag{T16.1}$$

$$S_i = 1 - e^{-Z_i} \tag{T16.2}$$

Partial for survivorship

$$\frac{\partial \hat{\iota}_i}{\partial F^*} = \begin{cases} 0, & i = 1 \\ e^{-Z_{i-1}} \left(\dfrac{\partial \hat{\iota}_{i-1}}{\partial F^*} - \hat{\iota}_{i-1} v_{i-1} \right), & i > 1 \\ \dfrac{e^{-Z_{i-1}}}{1 - e^{-Z_i}} \left(\dfrac{\partial \hat{\iota}_{i-1}}{\partial F^*} - \hat{\iota}_{i-1} v_{i-1} \right) - \hat{\iota}_{i-1} e^{-Z_{i-1}} v_i e^{-Z_i}, & i = A \end{cases} \tag{T16.3}$$

Partials for incidence functions

$$\frac{\partial \phi_e}{\partial F^*} = \sum_{i=1}^{\infty} f_i \frac{\partial \hat{\iota}_i}{\partial F^*} \tag{T16.4}$$

$$\frac{\partial \phi_q}{\partial F^*} = \sum_{i=1}^{\infty} \frac{w_i v_i S_i}{Z_i} \frac{\partial \hat{\iota}_i}{\partial F^*} + \frac{\hat{\iota}_i w_i v_i^2}{Z_i} \left(e^{-Z_i} - \frac{S_i}{Z_i} \right) \tag{T16.5}$$

Partial for recruitment

$$\frac{\partial R_e}{\partial F^*} = \frac{R_o}{\kappa - 1} \frac{\phi_E}{\phi_e^2} \frac{\partial \phi_e}{\partial F^*} \tag{T16.6}$$

Table 17: Statistical catch-age model using the Baranov catch equation and C^* and F^* as leading parameters.

Data

$$C_j = C_j^{\mathrm{US}} + C_j^{\mathrm{CA}} \tag{T17.1}$$

$$I_j, A_{i,j}, Q_{i,j,l} \tag{T17.2}$$

Parameters

$$\Theta = (C^*, F^*, M, \hat{a}, \hat{\gamma}, \bar{a}, \bar{\gamma}, \{\omega_t\}_{t=1}^{T-1}, \rho, \vartheta^2) \tag{T17.3}$$

$$\sigma^2 = \rho\vartheta^2, \quad \tau^2 = (1-\rho)\vartheta^2, \quad \sum_t \omega_t = 0 \tag{T17.4}$$

Unobserved states

$$N_{i,j}, B_j, E_j, F_j \tag{T17.5}$$

Initial states (t=1)

$$N_{i,j} = B_o/\phi_B \iota_i \tag{T17.6}$$

State dynamics (t>1)

$$E_j = \sum_i N_{i,j} f_i \tag{T17.7}$$

$$Z_{i,j} = M + F_j v_i \tag{T17.8}$$

$$\hat{C}_t = \sum_i \frac{N_{i,j} w_i F_j v_i \left(1 - e^{-Z_{i,j}}\right)}{Z_{i,j}} \tag{T17.9}$$

$$F_{j_{i+1}} = \ F_{j_i} - \frac{\hat{C}_j - C_j}{\hat{C}_j'} \tag{T17.10}$$

$$N_{i,j} = \begin{cases} \dfrac{s_o E_{t-1}}{1+\beta E_{t-1}} \exp(\omega_t - 0.5\tau^2) & a = 1 \\ \\ N_{t-1,a-1} \exp(-Z_{t-1,a-1}) & a > 1 \end{cases} \tag{T17.11}$$

$$B_t = \sum_a N_{t,a} w_a v_a \tag{T17.12}$$

Residuals & predicted observations

$$\epsilon_t = \ln\left(\frac{I_t}{B_t}\right) - \frac{1}{n}\sum_{t \in I_t} \ln\left(\frac{I_t}{B_t}\right) \tag{T17.13}$$

$$\hat{A}_{t,a} = \frac{N_{t,a}\dfrac{F_t v_a}{Z_{t,a}}\left(1 - e^{-Z_{t,a}}\right)}{\sum_a N_{t,a}\dfrac{F_t v_a}{Z_{t,a}}\left(1 - e^{-Z_{t,a}}\right)} \tag{T17.14}$$

Table 18: Likelihoods and priors used in the statistical estimation of Θ from Table 17.

Negative log-likelihoods

$$\ell(\Theta)_1 = \sum_{t=1}^{T-1} \left[\ln(\tau) + \frac{\omega_t^2 - 0.5\tau^2}{2\tau^2} \right] \tag{T18.1}$$

$$\ell(\Theta)_2 = \sum_{t \in I_t} \left[\ln(\sigma) + \frac{\epsilon_t^2}{2\sigma^2} \right] \tag{T18.2}$$

$$\begin{aligned} \ell(\Theta)_3 = & \\ & \sum_{t \in A_{t,a}} \sum_{a=2}^{A} \left\{ \ln(\varsigma) + \ln \left[\exp\left(\frac{-(P_{t,a})^2}{2\varsigma^2} \right) + 0.01 \right] \right\}, \\ & \text{where} \quad \varsigma = (\hat{A}_{t,a}(1 - \hat{A}_{t,a}) + 0.1/A)n, \\ & P_{t,a} = (A_{t,a} - \hat{A}_{t,a}) \end{aligned} \tag{T18.3}$$

$$\ell(\Theta) = \sum_{i=1}^{3} \ell_i \tag{T18.4}$$

Constraints

$$\kappa > 1.0 \tag{T18.5}$$

Posterior distribution

$$P(\Theta) \propto \exp[-\ell(\Theta)] p(C^*) p(F^*) p(M) p(\rho) p(\vartheta^2) \tag{T18.6}$$

C STAR panel requests

Request: Fmsy prior sensitivity. Shift the prior plus/minus 20%. Rationale: How sensitive is the management advice (e.g., Table 2 and 5) to the the prior.

The STAR panel requested a sensitivity analysis about the effect of the assumed prior distribution for F^*on the catch advice. To conduct this sensitivity analysis, I performed 3 additional assessments using the alternative prior distributions shown in Fig. 20. Three alternative distributions were explored: two where the same expected variance was assumed but the mean was plus or minus 20% of the assumed value, and a third distribution with the same expected mean but a larger assumed variance.

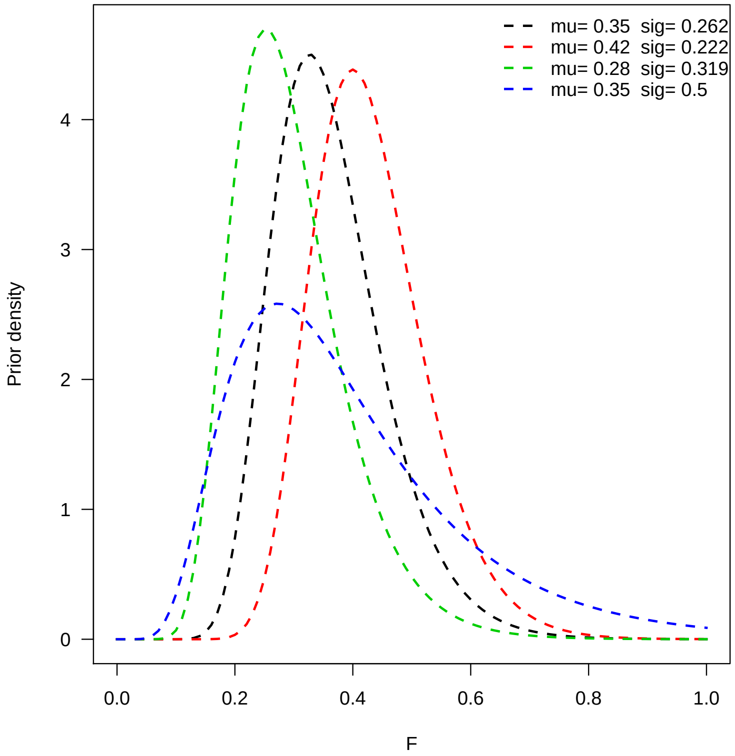

Figure 20: Alternative prior distributions for F^*. Note that the black distribution corresponds to the assumed distribution that was used to generate the catch advice.

Due to time constraints, the number of samples from the joint posterior distribution was reduced to 200,000 draws from which 5,000 systematic samples were taken. Overall, there was only minor differences in marginal posterior densities based on this reduced sample size and each run satisfied running median convergence diagnostics (Geweke statistic in the Coda package, R Development Core Team (2006)).

There is insufficient information in the data to resolve F^*, and the corresponding marginal posterior distribution for this parameter resembles the assumed prior density (Fig. 21)

The marginal posterior distribution for the catch advice based on the 40/10 control rule is sensitive to the assumed prior for F^*; increases in the prior mean for F^*result in increases in the median of the marginal posterior distribution for ABC. Estimates of $C^*$are somewhat insensitive to the assumed prior values; increases in the prior mean for F^*has very little effect on the marginal posterior mean for C^*.

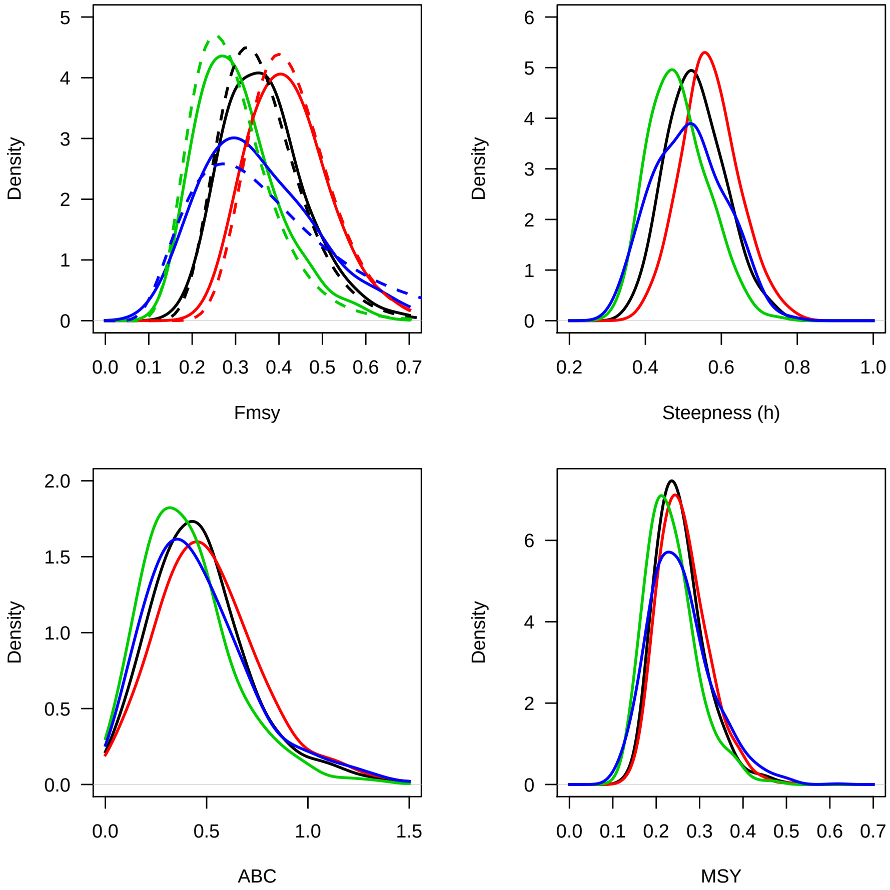

Figure 21: Sensitivity of the marginal posterior distributions for F^*, steepness, catch advice, and $C^*$to alternative priors on F^*. Note that the priors are shown for F^*only (dashed lines in the top left panel).

Estimates of spawning stock depletion are insensitive to alternative prior distributions for F^*(Fig. 22). Similarly, the lower bound estimates of the unfished spawning stock biomass is insenstive to the alternative priors for F^*. Also, there is little to no sensitivity of M to the alternative priors for F^*(Fig. 22). The median estimates of the SPR values associated with $F^*$are sensitive to the assumed prior distributions for F^*, where the mode of marginal distributions for $\mathrm{SPR}_{\mathrm{F}_{\mathrm{MSY}}}$ decreases as the mode of the prior distribution for F^*increases.

It is not surprising to see that the scaling parameters are insensitive to the rate parameter (i.e., $F^*$and M) as the parameterization of this model is designed to reduce the confounding between scale parameters (e.g., unfished biomass) and rate parameters (e.g., steepness).

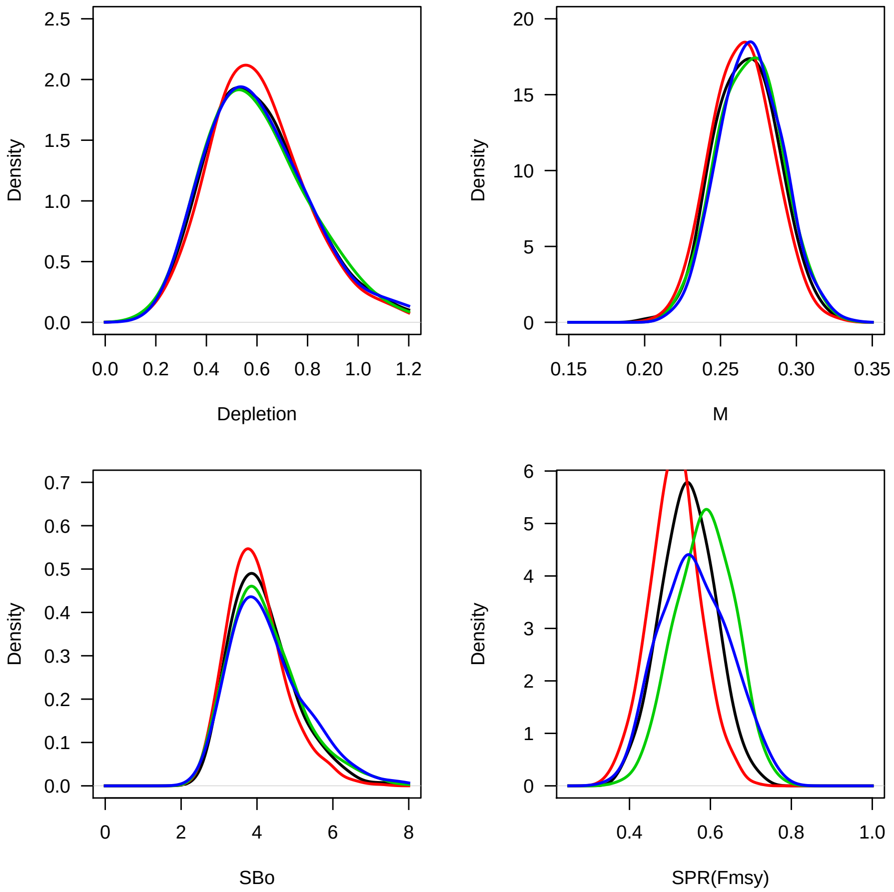

Figure 22: Sensitivity of the depletion estimates, unfished spawning stock biomass (SBo), instantaneous natural mortality rate (M) and the SPR vale at Fmsy to alternative prior assumptions about F^*.

Overall, the catch advice is relatively insensitive to alternative prior distributions for F^*(Fig. 23) for a given risk level. At most catches vary by 20,000 mt for any given risk level. Note also that the risk of overfishing calculation results in more conservative harvest policies in comparison to the cumulative distribution of catches produced by the 40/10 harvest rule (Fig. 24).

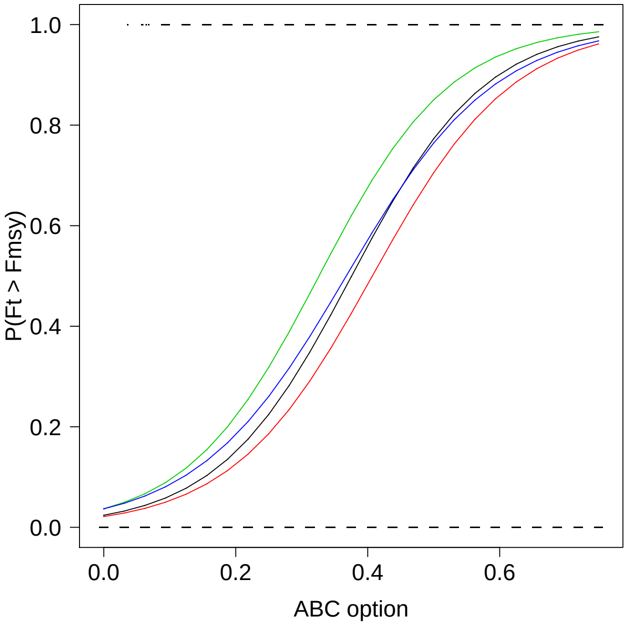

Figure 23: Probability of $F_{2008} \geq F^*$versus ABC option for the 2008 fishery for 4 alternative prior distributions for F^*.

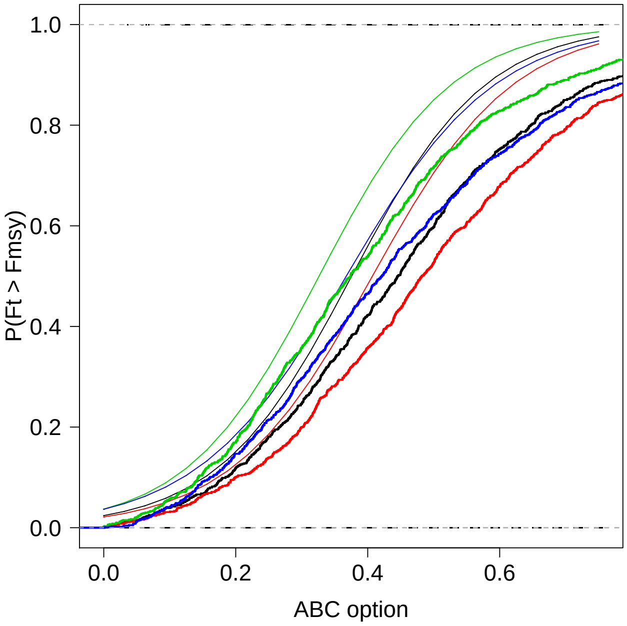

Figure 24: Probability of $F_{2008} \geq F^*$versus ABC option for the 2008 fishery for 4 alternative prior distributions for F^*, and the cumulative density functions of the catch advice produced by the 40/10 harvest control rule (thick lines).

D Retrospective plots

The following retrospective plots were presented at the STAR panel meeting. Note that scale of the age-1 recruits (Fig. 27 is substantially less in comparison to Fig. 15 due to a correction in the likelihood for the recruitment deviations pointed out by Ian Stewart and the STAR panel (the first draft omitted the bias correction term in negative log likelihood for the recruitment deviations). Estimates of age-1 recruits are now higher due to the higher estimates of the instantaneous natural mortality rate M.

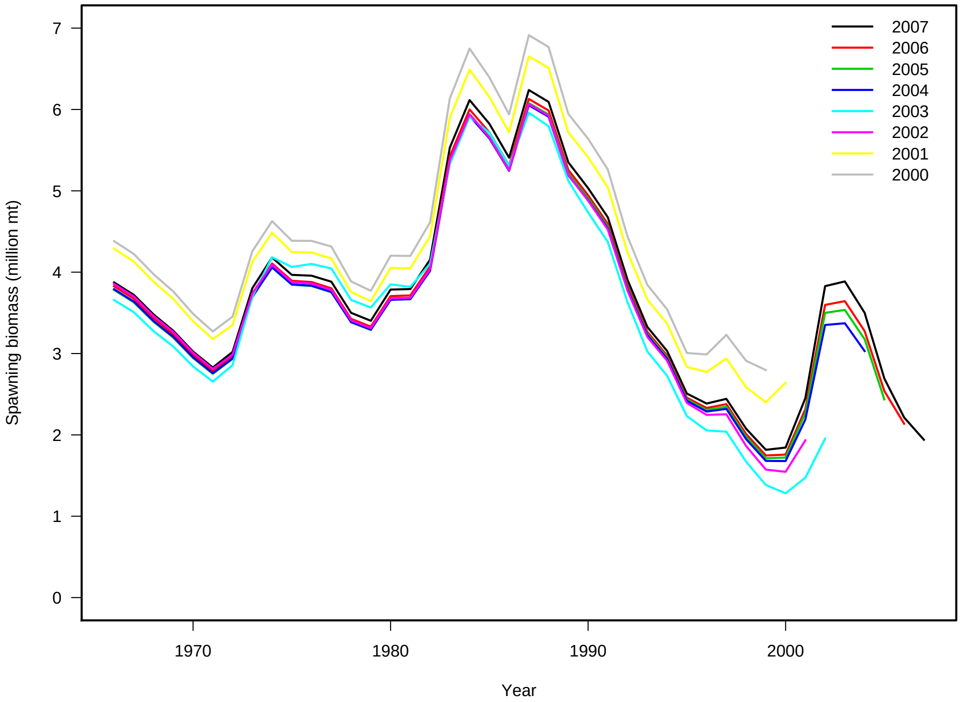

Figure 25: Retrospective pattern in the spawning stock biomass.

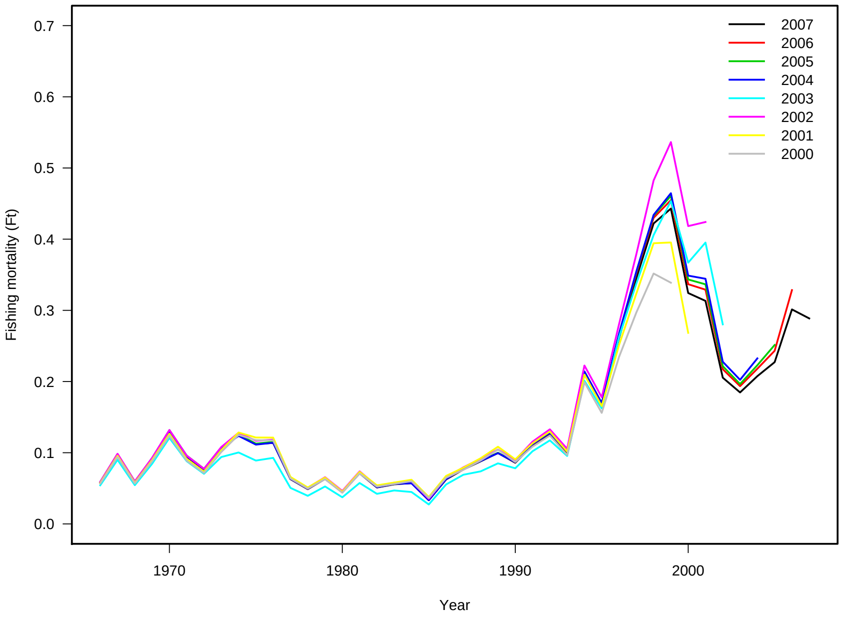

Figure 26: Retrospective pattern in the fishing mortality rates.

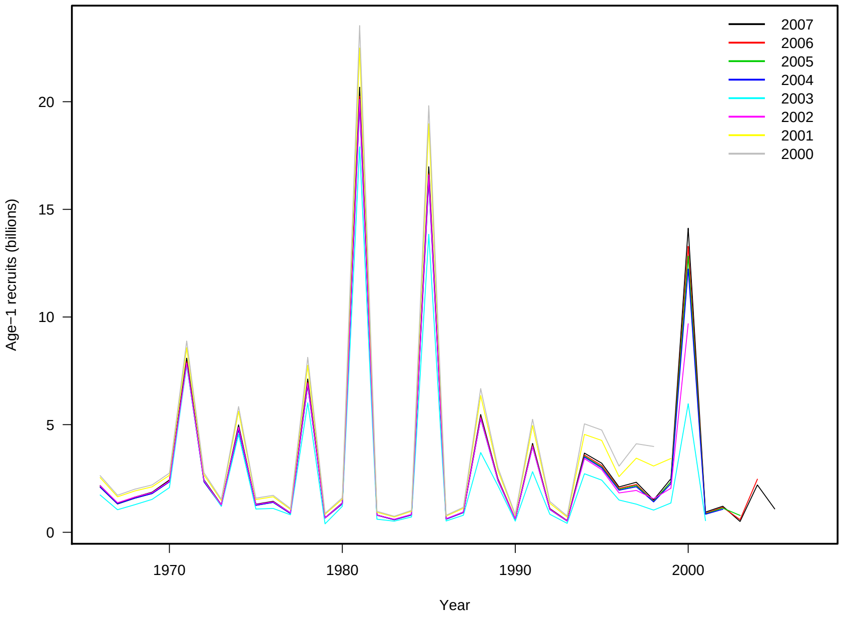

Figure 27: Retrospective pattern in age-1 recruits.